CONTENTS

CONTENTS 1
ACKNOWLEDGEMENTS 3
EXECUTIVE SUMMARY 4
LIST OF ABBREVIATIONS 9
LIST OF APPENDICES 11

Chapter

1 Introduction 12
Background of the Review 12
Terms of Reference of the Ad hoc Committee 12
Membership of the Ad hoc Committee 13
Mission of the Ad hoc Committee 14
Approach of the Review 15

2 The Current Mathematics Curriculum of Hong Kong 17
Primary 17
Secondary 18
Sixth Form 18
Revised Primary and Secondary Mathematics Syllabuses 20
Mathematics-Related Activities 21

3 Findings of the Research Studies 22
Research 1 – Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries 22
Research 2 – An Analysis of the Views of Various Sectors on the Mathematics Curriculum 23
4  Direction of Changes  
Mathematics and the School Curriculum  
The Role of Mathematics in the School Curriculum  
Looking Ahead  

5  The Anticipated Mathematics Curriculum  
Learning Dimensions in the Mathematics Curriculum  
Cross-level Interface in the Mathematics Curriculum  
Mathematics for Early Childhood Education  
Mathematics Curriculum for Post-Basic Education  

6  Implementation Strategies  
Schedule of Implementation of the Mathematics Curriculum at the Primary and Secondary Levels  
Assessment in the Mathematics Curriculum  
Catering for Learner Differences  
Quality of Mathematics Teachers  
Using Information Technology in Mathematics Education  

7  Summary of Recommendations  
Direction of Changes  
The Anticipated Mathematics Curriculum  
Implementation Strategies
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St Joseph’s College
The University of Hong Kong
The University of Hong Kong
The Chinese University of Hong Kong
The Chinese University of Hong Kong
CDC Committee on Mathematics Education (Chairman)

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CDC Mathematics Subject Committee (Secondary)
CDC Mathematics Subject Committee (Sixth Form)
CDC Applied Mathematics Subject Committee (Sixth Form)
CDC/HKEA Joint Working Party on Sixth Form Mathematics Syllabuses Revision
The Research Team of the University of Hong Kong
The Research Team of the Chinese University of Hong Kong

We are also grateful to any person who has expressed his/her views to us during the process of the review.
EXECUTIVE SUMMARY

There has been an increasing concern about the mathematics education in the school system against the structural changes in the Hong Kong society. The former CDC\(^1\) therefore set up an Ad hoc Committee to conduct a holistic review on the mathematics curriculum in July 1997 to examine the related issues and to make recommendations on the way forward.

To help the Ad hoc Committee to make recommendations to CDC at the end of 1999, two supportive research studies were respectively commissioned to the University of Hong Kong and the Chinese University of Hong Kong in 1998. The two studies were “Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries” and “An Analysis of the Views of Various Sectors on the Mathematics Curriculum”. The two studies were completed with reports compiled in mid-1999. Two press briefings were arranged on 16 December 1998 and 28 June 1999 respectively to publicize the progress of the holistic review of the mathematics curriculum and disseminate the findings from the two studies.

The Ad hoc Committee totally held 22 meetings from July 1997 to December 1999 and worked out 10 position statements to consolidate its viewpoints on major issues related to mathematics education. The statements served as a basis for exchanging ideas between the Ad hoc Committee and the relevant CDC subject committees. The position statements were reviewed and revised in the light of comments received. Eventually, these statements are incorporated into the Ad hoc Committee’s final report. Issues addressed in the report are summarized in the following paragraphs.

Mathematics and the School Curriculum

The Ad hoc Committee has summarized the importance and significance of mathematics education in the context of a dynamically changing environment. Mathematics is essential for everyone to become a responsible citizen of the modern age; a knowledgeable or skillful worker in all walks of life; or an expert or a professional of a particular field. The general aims of mathematics education are to develop our youngsters’ knowledge, skills, concepts, confidence and interest in mathematics to enable them to master mathematics, and more importantly to further develop related core competence, such as numeracy and logical reasoning throughout their lifetime. The Ad hoc Committee proposes that the curriculum content of the primary and secondary mathematics curricula should be developed to materialize their specific aims. Explicit statements for various aspects of the specific aims at the sixth form level and in each mathematics subject should be developed with reference to those of the primary and secondary mathematics curricula. This will help to form a holistic

\(^1\) The CDC has been re-structured in September 1999.
view of what mathematics education is and how mathematics education can contribute towards the aims of the school curriculum.

**Learning Dimensions in the Mathematics Curriculum**

The Ad hoc Committee considers that the mathematics curriculum should be designed according to a set of content-based learning dimensions so that learning objectives and students' progress can be structured and represented systematically from primary through secondary level. Mathematics learning should progress from concrete to abstract. Students need adequate prior experience with concrete objects before formal treatment of mathematical concepts. The progression within different dimensions is arranged so that prerequisite knowledge is sufficiently tackled. High order thinking skills should be incorporated with the content-based learning dimensions to form a reference grid in designing the future mathematics curriculum.

**Cross-level Interface in the Mathematics Curriculum**

The adopting of a formal treatment in teaching mathematics in some kindergartens has resulted in a continuity problem for students upon admission to Primary 1 as kindergarten learning is not assumed in the development of the primary mathematics curriculum. Some of the materials at Primary 5 & 6 and Secondary 1 are found overlapped. The poor continuation between Additional Mathematics and Advanced level Pure Mathematics and Advanced Supplementary level Mathematics & Statistics is also a concern to most senior level secondary teachers. The Ad hoc Committee therefore recommends that the mathematics curriculum at various levels of schooling should be smoothened to ensure coherence and continuity of the curriculum. To achieve the purpose, the mathematics learning at the kindergarten level should be activity-based, intuitive and simple while that in the primary and secondary schooling should be considered as an entity and should progress from concrete to abstract. The teaching of abstract mathematical ideas should be supported by students' concrete experiences at earlier stages as far as possible and teaching strategies should be progressively changed through different levels of schooling so as to cope with students' development. The mathematics curriculum across different levels of learning should be duly adjusted to suit the different abilities of students. The interflow between primary and secondary mathematics teachers should also be enhanced.

**Mathematics for Early Childhood Education**

The introduction of mathematics in the early childhood education is to help children enrich and supplement their informal mathematical experience in a meaningful way. It provides children with an opportunity to expose themselves to the experience of learning mathematics through various kinds of mathematical activities. It also gives children an opportunity to develop their abilities to apply mathematical concepts and skills in daily life situations. Since children at the pre-primary level are only expected to acquire basic skills
and concepts in mathematics, the Ad hoc Committee holds that they should not be formally assessed and no prerequisite academic knowledge should be expected of them on their admission to Primary 1. Heuristic methods of teaching should be adopted to help children develop an interest in mathematics learning. Thematic approach with integrated activities of learning should be used for the purpose of flexible curriculum integration.

**Mathematics Curriculm for Post-Basic Education**

For equality of access and reducing the labeling effect, the Ad hoc Committee does not agree to have curriculum differentiation in the years of general education. Mathematics should be a subject studied by all students. The curriculum should consist of foundation and non-foundation parts like the Secondary Mathematics Syllabus (1999) to cater for learner differences. At the upper end of the secondary schooling, different mathematics subjects, which may come from different combinations of modules and papers, could be developed to cater for the diversified needs of students. The Ad hoc Committee does not agree to stream students at an early age. On the other hand, opportunities for taking mathematics at the upper end of the secondary schooling should be allowed for all students.

**Schedule of Implementation of the Mathematics Curriculum at the Primary and Secondary Levels**

As the primary and secondary mathematics syllabuses were under revision during the process of review, the Ad hoc Committee also scrutinized the revisions critically. The Ad hoc Committee agrees to have the finalization of the revised primary mathematics syllabus in August 2000 so that the Ad hoc Committee's recommendations can be incorporated in the syllabus as far as possible. In this connection, the earliest implementation date of the revised syllabus will be deferred to September 2002. However, in the meantime, teaching materials on development of number and spatial sense and use of Information Technology (IT) in primary mathematics should be published to keep teachers of primary schools informed of the recent development of mathematics education. For the revised secondary mathematics syllabus, the Ad hoc Committee accepts that the major concerns, such as learning dimensions, catering for learning differences, enhancing of high order thinking skills and use of IT in mathematics, have been incorporated in the revised syllabus. The revised secondary mathematics syllabus was finalized in July 1999 and will be implemented at Secondary 1 in September 2001. The syllabus will then be reviewed together with the primary mathematics syllabus, if necessary, after considering the Ad hoc Committee's recommendations.

**Assessment in the Mathematics Curriculum**

The Ad hoc Committee accepts that assessment should be an integral part of the teaching-learning cycle. Assessment should play an important role in providing feedback for improvement in teaching and learning. Assessment of students' performance may take many forms and should be integrated with other classroom activities. It is agreed that the
design of learning objectives, learning activities and assessment tasks should be aligned to ensure that what is intended will be properly taught and successfully learned. The possibility of setting minimal competence for mathematics at various stages of schooling should be explored. The Ad hoc Committee also considers that assessment for high-stake purposes (such as placement and selection) should be played down so that interruption to normal teaching and learning in schools can be minimized.

Catering for Learner Differences

Learner differences are inevitable as different students have different sets of developed intelligence, abilities and educational experience. In spite of the difficulties teachers face in school, like crowded classroom and packed curriculum, the Ad hoc Committee holds that suitable measures could be taken to reduce the effect brought about by the learner differences of students. In addition, equal attention should be placed on both lower academic achievers and more able students. In the curriculum aspect, a flexible curriculum embracing foundation and non-foundation parts and enrichment content is a possible measure. Apart from the flexibility elements in the mathematics curriculum at the primary and secondary levels, curriculum differentiation at the upper end of the secondary schooling can be considered to cater for students’ different needs. In the school aspect, schools can adopt organizational arrangements like ability grouping in catering for the learner differences. Mathematics-related activities are also good means to cultivate the interest of students. In the classroom aspect, curriculum adaptation, relevant and realistic contextual tasks, and active and purposeful learning activities that allows individualized ways in the construction of knowledge are helpful. Since the success of these measures relies on teachers’ professional judgement, competent teachers are required to deal with the different mathematics abilities of students. In the assessment aspect, more emphasis should be laid on the assessment of minimal competence instead of the unnecessarily complicated problems. Moreover, a wide range of assessment activities is recommended for getting adequate information to organize students’ learning experiences.

Quality of Mathematics Teachers

The Ad hoc Committee is very concerned about the training of mathematics teachers. Effective implementation of the mathematics curriculum relies very much on the supply of knowledgeable and well-prepared teachers who are able to put the ideas of the mathematics curriculum into practice and realize the benefits as stated in the mathematics syllabuses. A good mastery of teaching methods as well as a strong mathematical background are important traits for a mathematics teacher. It is advisable to strengthen the subject specific component in Bachelor of Education courses. On the other hand, training of teaching methods should be provided to practising teachers with degrees in mathematics or related disciplines. The Ad hoc Committee generally agrees that an ideal qualification for a mathematics teacher, in the long run, is a bachelor’s degree in mathematics or related discipline together with Postgraduate Diploma in Education or Postgraduate Certificate of Education. Mathematics
teachers should see the importance of life-long education for teachers. Collegiate exchange among mathematics teachers should be encouraged.

Using IT in Mathematics Education

The Ad hoc Committee has examined the use of IT in teaching and learning mathematics and agrees that using IT in teaching and learning mathematics may bring about many benefits. IT can be used to enhance and extend mathematics learning experiences. Using IT in learning mathematics should encourage active student participation in exploratory and investigative activities. The future mathematics curriculum should focus on the effective use of information for problem solving - one of the principal reasons for studying mathematics. The Ad hoc Committee stresses that mathematics should be taught in its own right and with its own educational objectives for the information age. The Ad hoc Committee also realizes that IT should be cautiously used in classrooms. Teachers should act professionally towards choosing the most appropriate educational technology to benefit their students.
## LIST OF ABBREVIATIONS

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-level</td>
<td>Advanced level</td>
</tr>
<tr>
<td>AID</td>
<td>The Advisory Inspectorate Division</td>
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<tr>
<td>AS-level</td>
<td>Advanced Supplementary level</td>
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<tr>
<td>BA</td>
<td>Bachelor of Arts</td>
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<tr>
<td>BEd</td>
<td>Bachelor of Education</td>
</tr>
<tr>
<td>BSc</td>
<td>Bachelor of Science</td>
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<td>CDC</td>
<td>The Curriculum Development Council</td>
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<td>CDI</td>
<td>The Curriculum Development Institute</td>
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<tr>
<td>CE-level</td>
<td>Certificate of Education level</td>
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<tr>
<td>CUHK</td>
<td>The Chinese University of Hong Kong</td>
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<tr>
<td>ECR</td>
<td>Education Commission Report</td>
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<td>HK</td>
<td>Hong Kong</td>
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<tr>
<td>HKBU</td>
<td>The Hong Kong Baptist University</td>
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<tr>
<td>HKEA</td>
<td>The Hong Kong Examinations Authority</td>
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<tr>
<td>HKIEd</td>
<td>The Hong Kong Institute of Education</td>
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<tr>
<td>HKMO</td>
<td>The Hong Kong Mathematics Olympiad</td>
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<td>HKU</td>
<td>The University of Hong Kong</td>
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<tr>
<td>HOTs</td>
<td>The high order thinking skills</td>
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<tr>
<td>IMO</td>
<td>The International Mathematical Olympiad</td>
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<tr>
<td>IT</td>
<td>Information Technology</td>
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<tr>
<td>OUHK</td>
<td>The Open University of Hong Kong</td>
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<tr>
<td>PGCE</td>
<td>Postgraduate Certificate of Education</td>
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<td>PGDE</td>
<td>Postgraduate Diploma in Education</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
<td>--------------------------------------------------</td>
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<tr>
<td>PoS</td>
<td>Programme of Study</td>
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<tr>
<td>P.1, etc</td>
<td>Primary 1, etc</td>
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<tr>
<td>S.1, etc</td>
<td>Secondary 1, etc</td>
</tr>
<tr>
<td>TIMSS</td>
<td>The Third International Mathematics and Science Study</td>
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<tr>
<td>TOC</td>
<td>The Target Oriented Curriculum</td>
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<td>TTRA</td>
<td>The Targets and Target-Related Assessment</td>
</tr>
</tbody>
</table>
# LIST OF APPENDICES

<table>
<thead>
<tr>
<th>Appendix</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix 1</td>
<td>Executive Summary of Research 1 – Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries</td>
<td>55</td>
</tr>
<tr>
<td>Appendix 2</td>
<td>Executive Summary of Research 2 – An Analysis of the Views of Various Sectors on the Mathematics Curriculum</td>
<td>64</td>
</tr>
<tr>
<td>Appendix 3</td>
<td>Aims of the Revised Primary Mathematics Curriculum</td>
<td>71</td>
</tr>
<tr>
<td>Appendix 4</td>
<td>Aims of the Revised Secondary Mathematics Curriculum</td>
<td>72</td>
</tr>
<tr>
<td>Appendix 5</td>
<td>Examples of Learning Dimensions</td>
<td>73</td>
</tr>
<tr>
<td>Appendix 6</td>
<td>A Reference Grid for Learning Dimensions in the Future Mathematics Curriculum</td>
<td>74</td>
</tr>
<tr>
<td>Appendix 7</td>
<td>A Suggested Model of S.1 – 7 Mathematics Curriculum (Based on the Current Academic Structure)</td>
<td>75</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Background of the Review

1.1 Mathematics is a major subject in HK primary and secondary schools. The existing school mathematics curriculum comprises seven different subjects at different levels of schooling. Before the re-structuring of CDC in September 1999, the relevant syllabuses were under the ceilings of four different CDC subject committees, each of which is responsible for their own stage of schooling and scope of study. There were frequent calls from different mathematics education sectors for better co-ordination of the various mathematics syllabuses at primary, secondary and sixth form levels. These views were shared by members of various subject committees. The proposal of conducting a holistic review of the mathematics curriculum was therefore endorsed by CDC on 18 April 1997.

1.2 The Ad hoc Committee was subsequently set up by CDC in July 1997 to conduct the holistic review. The target of the review is to make recommendations to CDC on ways of enhancing continuity and intra-level coherence of the mathematics curriculum at various levels based on sound academic principles and practical demands. The Ad hoc Committee was expected to submit its final report to CDC by the end of 1999.

Terms of Reference of the Ad hoc Committee

1.3 The terms of references of the Ad hoc Committee are:
(a) To review the aims and objectives of mathematics education from P.1 to S.7.
(b) To review the mathematics syllabuses at different levels with particular attention to curriculum continuity and coherence.
(c) To initiate researches and surveys in support of the review.
(d) To propose the implementation strategies of the various syllabuses and to make recommendations on the need of teacher education and the provision of resources.

The four CDC subject committees are: CDC Mathematics Subject Committee (Primary), CDC Mathematics Subject Committee (Secondary), CDC Mathematics Subject Committee (Sixth Form) and CDC Applied Mathematics Subject Committee (Sixth Form).
To report the recommendations to CDC in due course.

Membership of the Ad hoc Committee

1.4 The membership of the Ad hoc Committee was as follows:

**Chairman**

Dr. WONG King-keung  
Member  
Hong Kong Airport Authority

**Vice-chairman**

Dr. LEUNG Yat-ming  
Chief Executive  
The Curriculum Development Institute, ED  
(until 5.2.1998)

Mrs LAI LAU Sui-kuen, Lily  
Acting Chief Executive  
The Curriculum Development Institute, ED  

Dr. CHAN Ka-ki, Catherine  
Chief Executive  
The Curriculum Development Institute, ED  
(since 7.9.1998)

**Members**

Mr. CHAN Kwok-wai  
Teacher  
Tai Po Government Primary School AM  
Teacher

Mr. CHAN Wai-chung  
Vice-Principal  
St. Francis Xavier’s School, Tsuen Wan

Prof. CHENG Shiu-yuen  
Head  
Department of Mathematics  
The Hong Kong University of Science & Technology

Mr. FUNG Chi-yeung  
Senior Lecturer  
Department of Mathematics  
The Hong Kong Institute of Education

Mr. HO Yue-shun  
Principal  
King’s College
Mr. LEE Siu-hok  
Principal  
SKH Tin Wan Chi Nam Primary School

Dr. SHEN Shir-ming  
Dean  
Faculty of Social Sciences  
The University of Hong Kong

Deputy Director  
School of Professional and Continuing Education  
The University of Hong Kong  
(since January 1999)

Mr. SIU Chung-leung  
Vice-Principal  
Shue Yan Secondary School

Mr. TSANG Kin-wah  
Principal Inspector  
The Advisory Inspectorate Division, ED

Mr. WAN Tak-wing  
Subject Officer  
The Hong Kong Examinations Authority

Dr. WONG Ngai-ying  
President  
The Hong Kong Association for Mathematics Education

Secretary  
Mr. LEUNG Shiu-keung  
Principal Inspector  
The Curriculum Development Institute, ED  
(until 31.3.1999)

Mr. KWAN Siu-kam  
Principal Curriculum Development Officer  
The Curriculum Development Institute, ED  
(since 1.4.1999)

Mission of the Ad hoc Committee

1.5 The mission of the Ad hoc Committee is to review holistically the mathematics curriculum of HK and develop a conceptual framework and guidelines for designing a coherent, challenging, balanced and flexible mathematics curriculum, with application in
real-life situations, which enables students to be developed as creative thinkers and helps equip them with suitable mathematical power for life-long learning so that they can remain to be competitive in the society of the 21st century. The Ad hoc Committee also enhances continuity and intra-level coherence of the mathematics curriculum at various levels and suggests viable implementation strategies to improve the quality of teaching and learning of mathematics.

**Approach of the Review**

1.6 During the period from July 1997 to December 1999, a total of 22 meetings were held. Apart from that, two briefing sessions on 16 December 1998 and 28 June 1999 to news media for publicizing the progress of the holistic review and one public forum on 16 December 1999 to primary and secondary school teachers for collecting feedback were conducted.

1.7 In conducting the review, the Ad hoc Committee focused its attention on the aims and objectives of the school mathematics education. It proposed to conduct researches and surveys to provide support and to substantiate the review. The Ad hoc Committee also expressed keen concern in the implementation strategies of future syllabus revisions, the need for teacher education and the provision of adequate resources.

1.8 The Ad hoc Committee has accomplished three major tasks. First, it gathered general information about teaching and learning of mathematics in HK and overseas. Academics from education institutions were invited to make presentations in the meetings. Second, the Ad hoc Committee put great emphasis on communicating and exchanging opinions with different CDC subject committees. Detailed discussions on recent development of mathematics education in primary and secondary schools were held with the CDC subject committees of the primary, secondary and sixth form levels. Third, the Ad hoc Committee agreed that supportive research would be needed to study views and expectation of different sectors on school mathematics as well as the world-wide trends of mathematics education. Two supportive research studies were designed. They were:

**Research 1:** Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries (The research team was led by Dr. Leung Koon-shing, Frederick of HKU.)

**Research 2:** An Analysis of the Views of Various Sectors on the Mathematics Curriculum (The research team was led by Dr. Wong Ngai-ying of CUHK.)

1.9 Based on the discussion of the issues, 10 position statements were worked out. In
the report, the issues are grouped under three categories, namely the direction of changes, the anticipated mathematics curriculum and the implementation strategies.

1.10 The Ad hoc Committee also expressed its views on the Primary and Secondary Mathematics Syllabuses which were being under revision during the process of review. The feedback collected from the consultations of these syllabuses together with the findings of the two research studies have been taken into account in finalizing the recommendations in the report.

1.11 In the report, Chapter 2 describes the current mathematics curriculum of HK while Chapter 3 gives the brief summaries of the findings of the two research studies. Their executive summaries can be found from Appendices 1 and 2 respectively while the full reports from the web site www.cdcdi.hk.linkage.net/cdi/maths/index.htm. Chapters 4 to 6 address the main areas of concern, the views and the proposed improvements of the Ad hoc Committee. Chapter 7 is a summary of the recommendations.
Chapter 2

The Current Mathematics Curriculum of Hong Kong

2.1 Currently there are 7 mathematics subjects at various levels of schooling. These subjects are Primary Mathematics (P.1-6), Secondary Mathematics (S.1-5), Additional Mathematics (S.4-5) and the sixth form subjects (S.6-7): AS-level Mathematics and Statistics, AS-level Applied Mathematics, A-level Pure Mathematics and A-level Applied Mathematics. The Primary and Secondary Mathematics are subjects for "all" while the others are studies to suit students of different abilities and needs.

Primary

2.2 The existing Primary Mathematics Syllabus was developed in 1983. Its emphasis is on how students learn rather than how teachers teach. It also aims at fostering a right learning attitude in students, developing students’ problem solving skills and their abilities to explore mathematical situation, discover relationships, analyze, reason and make judgement. It adopts a “spiral approach” so as to ensure that some topics are regularly revisited at different levels of schooling. However, it is criticized that the primary mathematics curriculum is too tight and mathematics is mostly learnt through intensive drilling particularly at the senior primary levels. In addition, there is an overlapping in the mathematics curricula at the pre-primary and P.1 levels.

2.3 TTRA, introduced in 1990 and modified and renamed as TOC in 1994, was expected to further enhance the spirit of the Primary Mathematics Syllabus. Targets on developing an ever-improving capability to communicate, inquire, reason and conceptualize mathematically and solve mathematical problems, appreciate the beauty of mathematics and apply mathematics to different contexts through the learning of the knowledge, concepts and skills/procedures in number, measures, algebra, shape & space and data handling are highlighted. However, it is worried that precision in TOC assessment might hamper learning and reinforce comparison. The heavy workload of teachers involved in performing continuous assessments (particularly for the HOTs) has also begotten teachers’ grievances.
Secondary

2.4 This current single teaching syllabus for all secondary schools was developed in 1985. It is a core subject in S.1-5 and it serves as a continuity of the development of numeracy begun in the primary schools. Similar to the primary curriculum, a "spiral approach" is adopted. Its main objectives include introducing to students a general sense of the pattern and power of mathematics, giving more emphasis to the nature and application of mathematics, and providing students with a basis for further work in science and mathematics. But more emphasis has been laid on treating this subject as a tool rather than a way of thought.

2.5 To meet the abilities of different groups of students, Tailoring Guide for the Secondary Mathematics Curriculum was issued in 1996. The most fundamental concepts (roughly two thirds of the whole syllabus) are identified as the "tailored part". The less able students are expected to complete only the "tailored part" in the 5 years of secondary education. To cope with this change, the HKEA has also changed the format of the CE-level mathematics paper in 1998. There are two sections. Section A only consists of questions on the tailored syllabus while Section B questions on the whole syllabus. The tailored syllabus of Secondary Mathematics is found to be well received by schools with less able students.

2.6 On the other hand, students in S.4 & 5 who are more able in mathematics may choose to take the subject Additional Mathematics which was developed in 1992 and aims to provide better foundations for Sixth Form mathematics and strengthen students' critical mathematical thinking. Hence, Additional Mathematics is usually taken by students of Science classes and not by those of Arts or Commerce classes. The main criticism of the subject is its overlapping in some of the content areas with the Secondary Mathematics, AS-level Mathematics & Statistics and A-level Pure Mathematics. This has brought some difficulties to teachers teaching the subjects concerned.

Sixth Form

2.7 In the wake of the recommendations of the ECR No. 2 regarding Sixth Form

2.8 AS-level Mathematics & Statistics is designed for S.6 & 7 students who do not plan to specialize in the study of physical sciences and engineering but wish to further their studies in mathematics beyond the CE-level. The subject aims at developing students' mathematical and statistical knowledge and providing them with a tool for the study of some other subjects. On the other hand, AS-level Applied Mathematics is designed for S.6 & 7 students who intend to continue their studies in science, engineering and technology. It aims at developing students' mathematical skills in solving real-life problems and providing them with a foundation of mathematical knowledge and skills required in scientific and technological studies.

2.9 Both the A-level Pure Mathematics and A-level Applied Mathematics are for S.6 & 7 students intending to continue their studies in science, engineering and technology. Previous knowledge of Additional Mathematics would definitely be an advantage to students studying these subjects although it is not explicitly required. The main objectives of A-level Pure Mathematics include developing students' understanding of basic mathematical concepts and their ability to use basic logical patterns and conventions of reasoning, and extending their mathematical skills in problem solving. On the other hand, A-level Applied Mathematics aims at developing students' mathematical skills in solving problems in mechanics and real-life problems, and providing them with a foundation of mathematical knowledge required in scientific and technological studies. It should be noted that the objectives and the teaching approaches of the A-level Applied Mathematics and its AS-level counterpart have no significant difference. In fact, their main difference is only the content coverage. The content of the latter subject is roughly half that of the former subject (mechanics is included in A-level Applied Mathematics but not in the AS-level counterpart), but the depth of treatment of both subjects remains more or less the same. A frequently voiced opinion by people is to eliminate one of the two subjects.

4 Rearrangements have been made to bring the syllabus of the AS-level Applied Mathematics in line with that of the A-level Applied Mathematics so that corresponding parts in the two syllabuses are identical. The syllabus was implemented in September 1998.
2.10 Parallel with the holistic review of the mathematics curriculum, the Primary and Secondary Mathematics Syllabuses were being under revision with a view to updating the framework to meet the challenge of societal change and the advancement of technology. The draft curriculum document of the primary mathematics had already been sent to schools for consultation, but the revision of the Secondary Mathematics Syllabus had just started. We had expressed our views to the revised syllabuses. The draft curriculum document of the secondary mathematics was then issued to secondary schools for consultation in 1998. Positive feedback was obtained from both the consultations and the respective reports were compiled.

2.11 To allow flexibility, the revised Secondary Mathematics Syllabus consists of foundation/non-foundation parts and enrichment topics while the revised Primary Mathematics Syllabus foundation and enrichment topics. Moreover, spare periods are provided for consolidation or enrichment activities to suit the teaching approaches and the standard of students in individual schools.

2.12 The revised Primary Mathematics Syllabus was originally planned to implement at P.1 in September 2001. However, to avoid the mismatch in work schedule with the holistic review of the mathematics curriculum, the implementation date has been postponed to September 2002. In this way, our recommendations will be properly considered and incorporated in the revised curriculum.

2.13 For the revised Secondary Mathematics Syllabus, major concerns in mathematics education, such as design of learning dimensions, catering for learner differences, enhancement of HOTs and use of IT in mathematics, have been incorporated. The revised

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5 The two reports are:
- 課程發展議會 (1997) • 《小學數學科課程大綱踟（2001 年）意見調查簡報》，香港：小學數學科科目
- 委員會
6 The foundation part of the syllabus is the essential part of the syllabus stressing the basic concepts, knowledge, properties and simple applications in real life situations. It represents the topics that all students should strive to master. The remaining topics of the syllabus constitute the non-foundation part of the syllabus.
7 The implementation date was originally planned to be September 2003 by the Ad hoc Committee. But the former CDC advised the Ad hoc Committee in its meeting held on 23 April 1999 that it should be implemented one year earlier.
curriculum will be implemented in September 2001. Since the curriculum is developed in line with our recommendations, no further major review seems necessary.

Mathematics-Related Activities

2.14 In HK, apart from the formal mathematics curriculum, mathematics-related activities play an important role in mathematics learning. We generally agree that well-chosen and organized mathematics-related activities should help to achieve the following objectives:
(a) To encourage co-operation and team spirit.
(b) To introduce new ideas and themes which are outside the existing mathematics curriculum.
(c) To broaden the scope of students' knowledge and facilitate them to appreciate the power and beauty of mathematics and be aware of its daily life application.
(d) To help students develop self-confidence, leadership and civic mindedness.
(e) To help students consolidate and widen the foundation in mathematics.

2.15 In schools, particularly secondary schools, Mathematics Club is usually established to co-ordinate the mathematics-related activities. The usual activities organized in schools include mathematical games, puzzles, mathematics trails, mathematics competitions/quizzes, mathematics workshops & projects, lectures, talks, film shows, book reviews, seminars, mathematics bulletin, tutorial classes, newspaper cutting and board display.

2.16 Apart from the mathematics-related activities organized within schools, both local and international activities are also held annually to promote students' interest in learning the subject. Local activities include Mathematics Trails, Mathematics Competition for Hong Kong Primary Schools, Hong Kong Primary Schools Mathematics Olympiad, HKMO, Poster Design Competition for HKMO, HKMO Mathematics Camp and Statistical Project Competition for Secondary School Students. International activities include Po Leung Kuk Primary Mathematics World Contest, Junior Mathematical Olympiad of China, Asian Pacific Mathematics Olympiad and IMO. However, it is found that students in HK seldom participate in mathematics-related activities.

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Mathematics Club is rarely established in primary schools. One of the possible reasons is that most of the primary schools are "half-day" schools. Teachers in either session do not have time and space to organize activities.
Chapter 3

Findings of the Research Studies

3.1 We proposed to conduct two supportive research studies to collect information about mathematics curriculum of other countries/regions and study different sectors’ views on mathematics education in primary and secondary schools of HK. The two studies were respectively commissioned to HKU and CUHK in 1998. The final reports were submitted in mid-1999. The summaries of the two research studies are as follows:

Research 1 – Comparative Studies of the Mathematics Curricula of Major Asian and Western Countries

3.2 The team members of the research study include Dr. Leung Koon-Shing, Frederick, HKU (principal investigator); Dr. Lam Chi-Chung, CUHK; Dr. Mok Ah-Chee, Ida, HKU; Mr. Wong Ka-Ming, CUHK and Dr. Wong Ngai-Ying, CUHK. The study consists of three components: a literature review, an analysis of curriculum documents, and a summary of the HK results in the TIMSS. The literature review includes the topics of student’s perception of mathematics and mathematics learning, student’s cognitive style and performance, goals of mathematics education, world trend of the mathematics curriculum, the anticipated change in the mathematics curriculum, etc.

3.3 33 curriculum documents from HK and 7 other countries were compared and the TIMSS Curriculum Analysis data were analyzed. It was found that:
(a) The revised Secondary Mathematics Syllabus (1999) in HK is generally in line with the worldwide trends.
(b) The HK mathematics curriculum attempts to strike a balance between process abilities (which are very much emphasized in the West) and basic skills and content (which are stressed in Asian countries).
(c) In HK, the introduction of topics into the curriculum is on average 2 years earlier than the international average.
(d) The textbooks in HK focus much of their attention on students’ performance of “knowing” and “using routine procedures”.
(e) A “canonical” curriculum is usually stipulated by the governments in Asian countries which is followed closely in schools.
(f) East Asian countries put a lot of emphasis on textbooks; on the contrary, western
countries are more flexible in their use of textbooks.

(g) Tracking for mathematics teaching is common, and there are various ways of implementing tracking in different countries.

(h) HK is probably the place with the least flexibility and choice in its mathematics curriculum.

3.4 The results of TIMSS, which are relevant to the theme of the study, are also summarized:

(a) HK students came fourth both in the 26 countries in grade four and the 41 countries in grade eight. They performed very well in routine problem solving but not so well at solving exploratory problems, and significantly worse in the TIMSS Performance Assessment where students were required to conduct some hands-on activities.

(b) Students in HK, like their counterparts in the rest of the TIMSS countries, found mathematics important, but they did not particularly like mathematics.

(c) Contrary to the common belief that students in East Asian countries attribute success more to hard work than to natural talent or ability, and that they attach a lot of importance to memorization, the TIMSS results indicate that students do not totally support these stereotypes. Teachers in HK however did not tend to believe in natural talent.

(d) Students in HK did not think that they did well in mathematics and in general, girls had a lower perception of their ability than boys.

(e) Compared to their counterparts elsewhere, HK students spent more out of school time doing mathematics homework, studying mathematics or attending extra mathematics lessons, especially at the primary school level.

3.5 The results show that HK students did extremely well in the TIMSS mathematics tests, but some students did not display the corresponding level of positive attitudes towards mathematics and some lacked confidence in doing mathematics. This may be a result of the Chinese culture stressing modesty, but it may also well be a result of the competitive examination system and a culture of lack of encouragement on the part of the teachers.

3.6 Readers can refer to Appendix 1 for the executive summary and the web site www.cdccdi.hk.linkage.net/cdi/maths/index.htm for the full report of the research.

Research 2 – An Analysis of the Views of Various Sectors on the Mathematics Curriculum

3.7 The team members of the research study include Dr. Wong Ngai-Ying (principal
investigator), CUHK; Dr. Lam Chi-Chung, CUHK; Dr. Leung Koon-Shing, Frederick, HKU; Dr. Mok Ah-Chee, Ida, HKU and Mr. Wong Ka-Ming, CUHK. During the research period, surveys (with questionnaires and interviews) were conducted on students, parents, teachers, university lecturers, curriculum planners and human resources personnel in the commercial sector to collect their views on the mathematics curriculum of HK.

3.8 The following gives a summary of the findings of the surveys. Readers can refer to Appendix 2 for a more detailed executive summary or the web site www.cdecodi.hk.linkage.net/cdi/ma for the full report of the research.

(a) Both students and parents showed high regard on mathematics.
(b) Different stakeholders held a positive view on the mathematics curriculum.
(c) Mathematics education should address a wider objective. HOTs should be addressed and teaching should provoke student thinking.
(d) The interest of students has to be maintained.
(e) The curriculum should be re-designed with epistemological and pedagogical considerations so as to strengthen thinking and conceptual understanding.
(f) The problem of learner differences has to be addressed including curriculum differentiation at senior secondary level.
(g) The idea of core and extended curriculum is worth further exploration.
(h) Continuation at all levels should also be secured. Teachers at various learning stages should have knowledge of the curriculum of other learning stages.
(i) Assessment and examination pressure should be carefully handled.
(j) The teacher is the key person to curriculum reform and he/she needs guidance and support on various issues including use of IT, enhancement of process abilities and curriculum tailoring.
(k) Pre-service and in-service teacher education should be strengthened.
(l) Collegiate exchange in the circle should be promoted.
(m) Different stakeholders should be well informed of the future curriculum changes to gain the supports.
(n) The workload of teachers should be carefully considered.
Chapter 4

Direction of Changes

Mathematics and the School Curriculum

4.1 Students need to develop a positive attitude towards learning in order to lead meaningful lives and remain competitive in the society of the 21st century. So far as the rapid growth of knowledge in the information age is concerned, mathematics has unquestionably become a necessity for every individual to contribute towards the prosperity of the society. Under the existing framework of the school curriculum, mathematics education should achieve specific aims in several aspects, such as intellectual, communicative, social and moral, personal and physical, and aesthetic.

The Role of Mathematics in the School Curriculum

4.2 Mathematics is essential for everyone in the modern society. Many of the developments and decisions made in industry and commerce, the provision of social and community services as well as government policy and planning etc., to some extent rely on the use of mathematics. However, the high technology like computers and calculators has profoundly changed the world of mathematics education. It is not only what aspects of mathematics are essential for learning, but also how mathematics is done and what attitude towards mathematics learning is fostered. Therefore, apart from mathematical content, processes and attitude are also essential core components for mathematics learning at various stages of schooling. Mechanical drilling and impractical topics are no longer essential and relevant.

4.3 The general aims of mathematics education are to develop our youngsters' knowledge, skills, concepts, confidence and interest in mathematics to enable them to master mathematics, and more importantly to further develop a positive attitude towards mathematics learning and related core competence, such as problem solving, communication, numeracy and logical reasoning throughout their life time. It should be an integral part of the general education and hence becomes a key learning area of the school curriculum of HK.

4.4 Students should acquire and interpret mathematical information for processing problems and tasks efficiently and accurately and be able to consolidate their mathematical
knowledge and develop procedures for solving the problems. This will accomplish mathematics learning as well as developing students' problem solving ability for life-long learning. In this way, students would progressively develop a capacity to solve problems and mathematics tasks with confidence.

4.5 In mathematics education, emphasis should also be placed on the development of positive attitude among students when going through the problem solving procedures. Mathematics learning can develop students' abilities to appreciate the beauty of nature and provide them with opportunities to enjoy the excitement and pleasure brought about by accomplishing mathematics tasks.

4.6 Mathematics provides a powerful means of communication. It can be used to present information in many ways like figures, tables, charts, graphs and symbols which can be further manipulated to deduce further information. Students have to develop ability to share information and ideas through describing, questioning, reasoning and explaining with mathematical concepts and knowledge. Through the communicative process, students will also expand their capacity from working individually to solving problems in a collaborative way.

4.7 Mathematics is the basis of scientific development and technology and an essence in many other fields. It is an analytical tool for studying other disciplines. Mathematics learning helps students enhance their understanding of the world. Therefore, students need to reach certain levels of mathematics standards for supporting advanced study in these disciplines and for further development in a majority of professions.

4.8 Mathematics should also be treated as an intellectual endeavour and a mode of thinking rather than a tool. It is a creative activity in which students can be fully involved and display their imagination, initiative and flexibility of mind. During the learning process, students should be encouraged to appreciate the beauty of mathematics. They should gradually develop their mathematics potential by forming their own conception of the discipline and realizing that mathematics plays a central role in the human culture.

4.9 In line with the above directions, the specific aims of the mathematics curricula at both the primary and secondary levels have been developed in the revisions of the Primary and Secondary Mathematics Syllabuses. Care has been taken to ensure that these two sets of specific aims, though self-contained, are complementary to each other. They are listed in Appendices 3 and 4.
4.10 We propose that the curriculum content of the primary and secondary mathematics curricula should be developed to materialize these aims. Moreover, explicit statements for various aspects of the specific aims at the sixth form level and in each subject should be developed with reference to those aims of the primary and secondary mathematics curricula. This will help to form a holistic view of what mathematics education is and how mathematics education can contribute towards the aims of the school curriculum.
Chapter 5
The Anticipated Mathematics Curriculum

Learning Dimensions in the Mathematics Curriculum

5.1 The idea of using dimensions (with other names, such as strands, stems, domains, attainment targets) is widely used in the design of various types of mathematics curricula in the world (see Appendix 5 for a few examples). Designing the mathematics curriculum in a framework of learning dimensions enables the learning objectives and students' progress to be structured and represented systematically from primary through secondary levels.

5.2 The mathematics curriculum should be designed so that content knowledge can serve as a means to develop an awareness of mathematics among students and they can gradually be able to conceptualize mathematically phenomena and problems met in daily life or in other disciplines. Furthermore, they can frame and formulate these problems in the mathematical language, to solve them with mathematical tools and then to make sense out of it in the mathematical way. In order to have this attained, the following points should be considered:

(a) The mathematics curriculum should be designed in such a way that mathematics learning progresses from concrete to abstract.

(b) The content in the mathematics curriculum should be arranged in such a way that students could get adequate prior experience with concrete objects before formal treatment of mathematical concepts. Abstract concepts should also be backed up by an abundance of mathematical and non-mathematical (daily-life) examples.

(c) Pre-mature exposure should be avoided at all costs.

(d) Prior experience of informal formulation of problems, such as sketches, diagrams, tabling of data, is necessary for formal problem solving.

(e) The arrangement of content should progress along various dimensions, for instance, starting from the manipulation of numbers and gradually transiting to the manipulation of symbols; from handling of shapes to describing location and then to spatial abilities in 2- and 3-dimensions; from the measurement with objects that can be directly measured to those which can only be measured indirectly and finally to those abstract measures such as measures of likelihood and central tendency; from handling information to problem solving and interpretations and inferences.
5.3 The pace of different dimensions is arranged so that pre-requisite knowledge is sufficiently tackled. Since the focus of learning and the particularity of learning different domains are emphasized, we think that the integration of various dimensions is not a viable arrangement among the dimensions. In fact, connection could be dealt with in teaching and integration should be left to application problems.

Recommendations
5.4 The mathematics curriculum should be designed according to a set of content-based learning dimensions so that learning objectives and students' progress can be structured and represented systematically from primary through secondary levels. However, it is not necessary to extend the use of dimensions in designing sixth form mathematics curriculum.

5.5 The mathematics curriculum should be designed in such a way that mathematics learning progresses from concrete to abstract. The content in the mathematics curriculum should be arranged to let students get adequate prior experience with concrete objects before the formal treatment of mathematical concepts. Abstract concepts should also be backed up by an abundance of mathematical and non-mathematical (daily-life) examples.

5.6 Since HOTs can only be developed through the learning of various mathematical knowledge in the content areas, they should be incorporated into the content-based learning dimensions to form a reference grid in designing the future mathematics curriculum (see Appendix 6).

5.7 Teachers should be aware of those mathematical learning experiences, like social mathematics, mathematics appreciation, history of mathematics, which are not easily defined within the framework.

Cross-level Interface in the Mathematics Curriculum

5.8 In the existing situation of HK, mathematics is a core subject from P.1 to S.5. At the senior secondary and sixth form levels, an Additional Mathematics syllabus and 4 mathematics syllabuses are respectively available for students who are interested in mathematics, while at the pre-primary level, preliminary exposure with mathematics is also experienced. Since kindergarten learning is usually not assumed, the primary mathematics is expected to start almost from scratch. However, formal treatments in teaching mathematics in some kindergartens are often found. This naturally results in a continuity problem for students upon admission to P.1.
5.9 At present, the school mathematics curriculum is more content-oriented. The mathematics curriculum of HK is a bit tight and difficult, and is sometimes criticized being not related to real life. Teaching, particularly at the senior primary, senior secondary and sixth form levels, when facing high-stake assessments, is examination driven. Rote learning and drilling exercises are the most commonly used means in class teaching.

5.10 The mathematics curriculum at S.I repeats some of the materials at P.5 and P.6 to allow some time for students to adapt to the change in the medium of instruction. The Additional Mathematics Syllabus is relatively long when compared with that of the Secondary Mathematics and is not in proportion with the allocated teaching time. Some overlapping with the Secondary Mathematics Syllabus is also found.

5.11 The poor continuation between the senior secondary and the sixth form curricula is also a concern to most senior level secondary teachers. Typical examples include Additional Mathematics, AS-level Mathematics and Statistics and A-level Pure Mathematics. The A-level Pure Mathematics curriculum is abstract, difficult and lengthy. According to the regulation of the public examination, Additional Mathematics is not a prerequisite of the A-level Pure Mathematics. However, it would be extremely difficult to study Pure Mathematics without Additional Mathematics. Consequently, students strive for studying Additional Mathematics though they do not have the ability. The target students of AS-level Mathematics and Statistics is not clear. Those who have taken Additional Mathematics find the AS-level Mathematics and Statistics too easy while Arts students find it too difficult. Moreover, the mathematics and statistics parts are not fully integrated.

5.12 The mathematics curriculum should be reviewed as a whole with particular attention paid to the smoothening of the interface at various levels of schooling to ensure coherence and continuity of the curriculum. Mathematics learning at the kindergarten level should be activity-based and as intuitive and simple as possible, and arouse students' interest towards learning and the curiosity they already have. In primary and secondary schooling, mathematics learning should go from concrete to abstract and mathematical skills should not be separated from its content. The practice of skills should go alongside with understanding. An abundance of concrete real-life and mathematical examples are necessary for the acquisition of more abstract notions. Primary mathematics should be considered as a foundation stage for the development of students' mathematics knowledge and skills while the secondary mathematics curriculum should serve as a continuation and extension of the primary mathematics curriculum. More emphasis should be put on training students' logical reasoning and critical thinking, and on developing students' mathematical concepts rather than on developing the computational skills only.
5.13 The mathematics education at the primary and secondary levels (including teaching content, teaching strategies and assessment) should be considered as an entity. Great care and measures should be taken to ensure cross-level linkage within different dimensions at the primary and secondary levels. Connection across dimensions should also be considered. The structure of the mathematics curriculum at the upper end of the secondary schooling should be reorganized to ensure continuation and suit the different future mathematics needs of students.

Recommendations

5.14 Different levels should be considered as a continuous learning process. Objectives across various levels should be coherent and allow the development of mathematical concepts from concrete to abstract. The learning objectives and students’ progress should be structured and systematically represented within and across levels.

5.15 The acquisition of process abilities should be allied with the learning of mathematical content. Topics in the mathematics curriculum should be arranged into progressive learning dimensions so that students’ cognitive development in mathematics can be enhanced in a more coordinated way and the relevance of students ensured. For instance, the teaching of abstract mathematical ideas should be supported by students’ concrete experiences at earlier stages as far as possible. Students need time to play around with concrete objects before proceeding to more abstract notions at the senior level.

5.16 The mathematics curriculum should be developed as a whole, but it should be duly adjusted across different levels of learning to cater for the different abilities of students. It is up to teachers’ discretion to re-visit some topics when needed. However, it should be noted that though the spiral approach has its strong points, teaching too many topics in a single year and making learning fragmented should be avoided. In addition, some measures like organizing bridging programmes should be taken to ensure that students of different abilities can follow. In the curriculum development process, a flow chart indicating the inter-relation of topics of different mathematics curricula at different levels is recommended to ensure continuity.

5.17 Teaching strategies should be progressively changed through different levels of schooling, say from concrete to abstract, so as to cope with students’ development. Teaching at P.1 should selectively adopt a thematic approach. Similarly, investigational work is encouraged at all the primary levels and should be continued in secondary schools.
5.18 Continuity should not be restricted to the curriculum document level. Teachers at different levels should be well informed on what is going on at the others. Measures should be taken to reinforce interflow between primary and secondary mathematics teachers.

Mathematics for Early Childhood Education

5.19 The aim of the early childhood education is to provide children with a balanced development in the moral, intellectual, physical, social and emotional aspects. The curriculum should include elements assisting young children to adapt themselves to social life and to understand the relationships between the individuals and society. Children should be educated in a natural and pleasant environment through various theme-related learning activities involving life experiences, for example, role-play and simulation games.

5.20 As children have already acquired some mathematical knowledge informally through their daily life experiences before entering schools, the introduction of mathematics in early childhood education is to help children enrich and supplement their informal mathematical experience in a meaningful way. It provides children with an opportunity to expose themselves to the experience of learning mathematics, which would be beneficial to them in learning mathematics at the primary level in later years. It also gives children an opportunity to develop their ability to apply mathematical concepts and skills in daily life situations. However, the formation of early mathematical concepts is only part of the general learning activities in the pre-primary curriculum. It should be conducted in integration with other learning activities. We stress that undue emphases on subject teaching should be avoided. A formal approach in mathematics teaching and learning is undesirable at this stage.

5.21 Mathematics in the early childhood education should introduce to children the basic knowledge in mathematics according to their ages, experiences and interests through various kinds of mathematical activities. The objectives of introducing early mathematical concepts are to

(a) stimulate children's interest and motivation in learning mathematics and cultivate in children a positive attitude through learning mathematics;

(b) help children develop their concentration and memory, observational and analytical abilities, and problem solving skills; build up the preliminary concepts of number, shape, space, data handling, time, computation and measurement; and develop their ability to communicate mathematical ideas and experiences.
Recommendations

5.22 Since children at the pre-primary level are only expected to acquire basic skills and concepts in mathematics, they should not be formally assessed and no prerequisite academic knowledge should be expected of them on their admission to P.I. Heuristic methods of teaching should be adopted to help children foster an interest in learning mathematics and develop an inquiry mind. Thematic approach with integrated activities of learning should be used for the purpose of flexible curriculum integration.

5.23 Mathematical concepts should be introduced according to the developmental stages of children, their interests and needs, by using simple language, and through learning activities which involve manipulation of objects and relate to children's experience. A mathematically enriched environment is essential for the children to work in. Teachers should see that there is an ample amount of relevant materials available for the children to manipulate and experiment with.

5.24 Children should be given the opportunities to explore, discover and develop mathematical ideas. Importance should be given to the process of learning mathematics rather than the outcomes of the activities. Flexibility should be allowed in planning teaching programmes and designing learning activities to cater for the learner differences, and incidental teaching should be adopted.

5.25 Children should be encouraged to discuss and describe their daily encounters in an appropriate mathematical vocabulary. Mathematical jargons like statistics, sum and difference, etc. which are beyond children's understanding should not be used. Over-teaching, premature exposure to mathematical knowledge and abstract concepts should be avoided as they may hamper children's development of learning mathematics.

Mathematics Curriculum for Post-Basic Education

5.26 As students move along the grade levels, they use mathematics at different extent. This is particularly pronounced at the upper end of the secondary schooling. Students may go for further studies in different disciplines such as science, technology, commerce and social sciences and some may join the work force in different professions.

5.27 In the existing situation, students in S.4 and S.5, who are more able in mathematics, may choose to take the subject Additional Mathematics which aims to provide better
foundations for studying Sixth Form mathematics and strengthen students' critical mathematical thinking. Since some of the content areas in this subject has overlapped with those of Secondary Mathematics, AS-level Mathematics & Statistics and A-level Pure Mathematics, previous knowledge of Additional Mathematics would definitely be an advantage to students studying the AS-level and A-level mathematics subjects although it is not explicitly required. This has not only brought some difficulties to teachers teaching the subjects concerned but has also generated an unnecessary inclination among students in taking Additional Mathematics even they do not have the competence.

5.28 Moreover, the similarity between the two Applied Mathematics subjects, the lengthiness of A-level Pure Mathematics and the unclear target students of AS-level Mathematics & Statistics have begotten grievances from the school teachers.

5.29 We hold that the mathematics curricula at the senior secondary and the sixth form levels have to be re-structured. The re-structuring should achieve the following objectives:
(a) To provide appropriate learning experiences for students of different abilities and orientation.
(b) To rest upon strong theoretical foundation including the pedagogical consideration and the consideration of the knowledge structure of mathematics itself.
(c) To ensure continuation and to avoid overlapping between the senior secondary and sixth form levels in mathematics.
(d) To broaden the possibility of studying mathematics at the sixth form level for all students.
(e) To prepare students for future education along different tracks.

5.30 We do not agree to have curriculum differentiation in the years of general education. Instead, mathematics should be a subject studied by all students. This subject should enable students to develop a mathematics literacy for the information age, the basic competence in mathematics and the foundation mathematics needed for a knowledgeable citizen. The identification of foundation and non-foundation parts is good enough to cater for learner differences. More details are provided in Chapter 6.

5.31 To cater for the needs of different students at the upper end of the secondary schooling, a number of subjects, which may come from different combinations of modules and papers, should be designed for students who aim to further their studies in the mathematics related fields. Streaming students at an early age should be avoided and opportunities for taking mathematics at the senior levels should be allowed for all students.
A Proposed Model of the Mathematics Curriculum

5.32 Based on the assumption that there is no change in the academic structure of the secondary schooling (that is, there are 5 secondary levels and 2 sixth form levels in the educational system), we recommend to have only one mathematics curriculum at the senior secondary levels with the subject Additional Mathematics deleted for the purpose of achieving equality of access and for reducing the labeling effect. This "general mathematics curriculum" should consist of foundation and non-foundation parts together with enrichment topics like the revised secondary mathematics curriculum to cater for the learner differences.

5.33 At the sixth form level, we propose to offer 6 modules: 4 on mathematics (i.e. Math1-Math4) and 2 on statistics (i.e. Stat1-Stat2). The depths and breadths of these modules are described in the following table:

<table>
<thead>
<tr>
<th>Module</th>
<th>Similar in depth and breadth to</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math1</td>
<td>The mathematics component of current AS-level Mathematics &amp; Statistics</td>
</tr>
<tr>
<td>Math2</td>
<td>Current CE Additional Mathematics minus Math1</td>
</tr>
<tr>
<td>Math3 + Math4</td>
<td>Current A-level Pure Mathematics minus current CE Additional Mathematics</td>
</tr>
<tr>
<td>Stat1</td>
<td>The statistics component of current AS-level Mathematics &amp; Statistics</td>
</tr>
<tr>
<td>Stat2</td>
<td>The statistics component of current A-level Applied Mathematics minus Stat1</td>
</tr>
</tbody>
</table>

Note: Some mathematics topics in the current A-level Applied Mathematics such as differential equations and numerical methods will be absorbed into the 4 modules on mathematics.

5.34 The recommended combinations of these 6 modules are:

<table>
<thead>
<tr>
<th>Subject</th>
<th>Modules</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS-level Mathematics</td>
<td>Math1+Math2</td>
</tr>
<tr>
<td>AS-level Statistics</td>
<td>Stat1+Stat2</td>
</tr>
<tr>
<td>AS-level Mathematics &amp; Statistics</td>
<td>Math1+Stat1</td>
</tr>
<tr>
<td>A-level Mathematics &amp; Statistics</td>
<td>Math1+Math2+Stat1+Stat2</td>
</tr>
<tr>
<td>A-level Mathematics</td>
<td>Math1+Math2+Math3+Math4</td>
</tr>
</tbody>
</table>

Note: Students may choose any one of these five subjects or the combination "A-level Mathematics + AS-level Statistics".
A diagrammatic representation of the model is shown in Appendix 7.

5.35 No module should be counted more than once in the public examination concerned. For example, students are not allowed to choose AS-level Statistics and AS-level Mathematics & Statistics at one sitting as the module Stat1 is common to both papers. On the other hand, students taking the modules Math1, Math2, Stat1 and Stat2 should pre-determine that they are counted as one A-level Mathematics & Statistics subject or as two AS-level subjects (i.e. AS-level Mathematics and AS-level Statistics). To avoid confusion, schools should consider offering subjects instead of modules. We believe that the above model can be easily adjusted even if there are changes in the academic structure of the educational system.
Chapter 6

Implementation Strategies

Schedule of Implementation of the Mathematics Curriculum at the Primary and Secondary Levels

6.1 Both the primary and secondary mathematics syllabuses were under revision when the Ad hoc Committee was established in 1997. According to the original work plan, the two revised syllabuses would be finalized in mid-1999 and implemented in September 2001. Since we planned to submit our review report in late 1999, the schedule of implementation was re-adjusted to incorporate our recommendations.

The Revised Primary Mathematics Syllabus

6.2 Currently most primary schools are implementing the TOC PoS for Mathematics Key Stages 1 and 2 (1995) with reference to the Primary Mathematics Syllabus (1983). As the Syllabus (1983) was only slightly adjusted during the writing of PoS, some topics were considered out-dated (such as the technique on the computation of large numbers is phased out as calculators can be used instead). The need for revision to meet societal needs was obvious. Furthermore, as teachers have to refer to the PoS and the Syllabus (1983) simultaneously for details of learning targets, teaching content and strategies of individual topics, revising the primary mathematics syllabus to produce a unified syllabus document will certainly be welcomed by teachers.

6.3 In 1995, the Primary Mathematics Syllabus was reviewed and revised to incorporate the TOC elements contained in the PoS to form a revised syllabus. In mid-1997, a revised syllabus framework was developed and issued for public consultation. The proposal was well received by the teachers. It was originally planned to finalize the revised syllabus by mid-1999 and implement in P.1 in September 2001. The implementation date is chosen because by that time, most primary schools will have completed the TOC mathematics programme for a 6-year cycle.

6.4 To avoid the mismatch in work schedule with the holistic review, the finalization of the revised Primary Mathematics Syllabus is re-scheduled to August 2000. In this way, our recommendations could be properly considered and incorporated in the revised syllabus. The arrangement will also allow time for the CDC Committee on Mathematics Education to
evaluate the implementation of the PoS in a complete 6-year cycle in 2001. Experiences
drawn from the implementation will be examined and made available for improving the
revised syllabus.

6.5 In the meantime, teaching materials on the development of number, spatial sense and
the use of IT in primary mathematics should be published to keep primary schools informed
of the recent development of mathematics education.

The Revised Secondary Mathematics Syllabus

6.6 The former CDC Mathematics Subject Committee (Secondary) revised the Secondary
Mathematics Syllabus with a view to updating the 1985 syllabus. Informal consultations
with professional organizations and sponsoring bodies were conducted in 1997. In mid-
1998, a revised syllabus framework was developed and issued for public consultation. The
design aims to fit in the curriculum needs of students who have undergone 6 years of TOC
programme in their primary schools. Major concerns in mathematics education, such as
design of learning dimensions, catering for learning differences, enhancing of HOTs and the
use of IT in mathematics, have been incorporated in the revised syllabus.

6.7 The revised syllabus was finalized in July 1999 and will be implemented in
September 2001 as planned. The implementation date is chosen because the first batch of
students undergoing the TOC programme will move on to S.1 by that time.

Summary of the Programme of Work of Syllabus Revision

6.8 The following programme is made under the assumption that the present education
system is unchanged. If there are changes in the education system, this schedule will be
adjusted accordingly.

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preparation of teaching materials for primary mathematics</td>
<td>since 1999</td>
</tr>
<tr>
<td>Finalization of revised Secondary Mathematics Syllabus</td>
<td>July 1999</td>
</tr>
<tr>
<td>Ad hoc Committee's recommendations</td>
<td>Dec. 1999</td>
</tr>
<tr>
<td>Finalization of new Primary Mathematics Syllabus*</td>
<td>Aug. 2000</td>
</tr>
<tr>
<td>Implementation of revised Secondary Mathematics Syllabus in S.1</td>
<td>Sept. 2001</td>
</tr>
<tr>
<td>Implementation of new Primary Mathematics Syllabus* in P.1</td>
<td>Sept. 2002</td>
</tr>
</tbody>
</table>

(* The new syllabus will be designed according to our recommendations, if necessary.)

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9 The organizations and sponsoring bodies consulted were the HK Association for Science and Mathematics
Education Ltd., the HK Association for Mathematics Education, the HK Buddhist Association, Po Leung Kuk,
Tung Wah Groups of Hospital, Sheng Kang Hui and Diocesan Schools Association.
Recommendation

6.9 We suggest the CDC Committee on Mathematics Education to consider our recommendations during the review of the Primary Mathematics Syllabus. At the same time, the content of the Secondary Mathematics Syllabus can be re-adjusted, if deemed necessary.

Assessment in the Mathematics Curriculum

6.10 Assessment can be used for a variety of purposes such as evaluating the teaching effectiveness, diagnosing the learning difficulties of students, etc. We have discussed the role of assessment in the mathematics curriculum and hold that assessment is a process of gathering information and interpreting result. It should be an integral part of the teaching-learning cycle. The evidence collected from the assessment activities is an important feedback from students and should be used for students to improve their learning and for teachers to adjust their teaching strategies and pace.

6.11 The complexity of learner performance cannot be described by a single set of scores or a single type of assessment activities. Both formative and summative assessments are necessary for providing a comprehensive profile of student performance. Evidence of learning should be collected through various modes of assessment activities to reflect the students' achievement in mathematics. However, it should be understood that not every learning outcome can be readily assessed and we should let different stakeholders understand it as well. The HOTs are difficult to be assessed by traditional written tests.

6.12 We agree that schools need to formulate their assessment policy according to their culture, teachers' experiences, learners' needs and interests. It is also agreed that the design of learning objectives, learning activities and assessment tasks should be aligned to ensure what is intended will be properly taught and successfully learned.

6.13 The assessment of students' performance should not only focus on paper-and-pencil tests. It may take many forms and should be integrated with other classroom activities. Examples include project work, class discussions, oral presentations and observations of students' performance during lessons. However, if this kind of school-based assessment is to be carried out in public examinations, piloting will deem to be most desirable and care must be taken to avoid the hindrance of students' performance by their available resources and backgrounds. The pros and cons should be considered carefully.
<table>
<thead>
<tr>
<th>Pros</th>
<th>Cons</th>
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<tbody>
<tr>
<td>- More information is available to the end users.</td>
<td>- Differences in assessment standards among schools exist which may lead to low reliability.</td>
</tr>
<tr>
<td>- The needs of different students may be catered for.</td>
<td>- Authenticity of students' work may sometimes be doubtful.</td>
</tr>
<tr>
<td>- This would be of use to those students who might do the external examination uncharacteristically poor but perform well in other areas such as project work.</td>
<td>- Continual pressure on students and teachers would be created within the whole assessment period.</td>
</tr>
<tr>
<td>- This would change the school examination culture whereby undue emphasis would not be given to the preparation of external examinations.</td>
<td>- Extra workload would impose upon teachers who have already been overwhelming with teaching and administrative duties.</td>
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</table>

6.14 Minimal competence, which is the pre-requisite to the learning of mathematics at the next stage, is needed to help teachers report student performance in terms of the basic knowledge, concepts and skills acquired.

Recommendations
6.15 The possibilities of setting minimal competence for mathematics at various stages of schooling should be explored.

6.16 We consider that assessment for high-stake purposes (such as placement and selection) should be played down to reduce over-drilling to students and minimize interruption to normal teaching and learning in schools. Assessment should be used as a means to collect feedback from students to improve teaching and learning.

Catering for Learner Differences

6.17 With the introduction of the universal education in HK, the issue of learner differences at all levels is increasingly concerned. As a subject for all, the problem is especially marked in mathematics. Different students have different sets of developed intelligence, abilities, personality traits and educational experience. Hence, learner differences are inevitable as students learn with different learning styles in mixed ability
classes. Furthermore, research suggests that learner differences in mathematics begin to widen as students progress to higher levels. The mismatch between the curriculum and individual learning styles is considered as one of the causes for the drop of students' interest in learning mathematics. Therefore, the mathematics curriculum should be reviewed in order to cater for the diversified needs and different abilities of students since developing the potential of students, maximizing their learning and nurturing their uniqueness are considered important in school education.

6.18 The needs of students at both ends of the ability scale are equally important. Opportunities to learn should be maximized for all students. That is to say, attention should not be placed only on academically lower achievers. The needs of the more able students should also be catered for. More resources should be provided to strengthen the existing institution for the gifted and those related mathematics activities. However, every means should be taken to prevent labeling effect.

6.19 We understand the difficulties teachers face in organizing learning activities in crowded classrooms under packed curriculum while students with different abilities have to compete in the same assessment system. Although constraints exist in the current educational context, suitable measures could be taken in planning educational experiences to reduce the effect brought about by the learner differences of students.

6.20 Based on the educational beliefs and identified constraints, the following measures in catering for learner differences could be taken in different aspects.

Curriculum Aspect

6.21 A general mathematics curriculum should be developed to provide all students in the years of general education with all necessary knowledge, concepts, skills and attitudes essential for a knowledgeable citizen of the modern age and the mathematical power, such as reasoning, for life-long learning. As a subject for all, students can acquire adequate fundamental knowledge of mathematics to support their fields of study. They should be equipped with the core competence necessary for the daily life and workplace on one hand, and be able to enjoy and appreciate the excitement and the beauty of mathematics on the other.

6.22 It is important that the mathematics curriculum should be flexible enough to cater for a wide spectrum of abilities of students. By identifying the foundation and non-foundation parts of the mathematics curriculum, teachers can select and organize relevant learning experiences according to students' abilities. Enrichment content could be provided for more
able students while content could be trimmed down and adapted for lower academic achievers.

6.23 In line with the Tailoring Guide for the Secondary Mathematics Curriculum (1996), the foundation part should
(a) be the minimum body of learning for every student,
(b) contain different components that constitute a coherent curriculum,
(c) emphasize on important knowledge, concepts and skills.
All teachers should try their best to help their students master this part.

6.24 As the content in primary mathematics curriculum is relatively fundamental, the foundation components in primary mathematics curriculum are relatively substantial. On the other hand, the non-foundation components should increase progressively in the years of basic education. Measures, such as identifying the foundation and non-foundation components, allocating spare time periods, facilitating curriculum adaptation, can also be considered.

6.25 As learning content and objectives in secondary mathematics curriculum are arranged in terms of key stages instead of levels, teachers can judge for themselves their own sequences of learning units by basing on the needs of their students.

6.26 Apart from incorporating the flexibility elements into the mathematics curriculum at the primary and secondary levels, curriculum differentiation at the upper end of the secondary schooling is adopted to cater for students' different needs. Students should be given opportunities for further studies as well as equal access to different combinations of modules according to their interests and needs. At the same time, students should be equipped with knowledge and skills necessary for further studies in tertiary education and application in the technological society. The current mathematics curriculum for the upper end of the secondary schooling should be re-organized, as stated in the subtitle “Mathematics Curriculum for Post-Basic Education” in Chapter 5, to achieve the purpose.

School Aspect
6.27 According to the needs, interests and abilities of students, schools can adopt organizational arrangements, such as ability grouping, as well as instructional and curricular arrangements (such as remedial teaching, integrated learning and enrichment activities), in catering for learner differences.

6.28 Mathematics-related activities like mathematics clubs, quizzes, competitions, games,
projects and workshops are good means both to cultivate the interest of students and to provide students with learning experiences through informal curriculum. Research suggests that students' participation of such activities is beneficial to students' learning. Through enhancement and consolidation activities, more able students can broaden their exposure in mathematics while lower academic achievers can consolidate their concepts.

**Classroom Aspect**

6.29 It is impractical for teachers to accommodate to the learning styles of all students in every lesson. However, active and purposeful learning activities, such as project work, worksheets, graded and optional class exercises, suggested leisure reading, that allows individualized ways in the construction of knowledge is definitely helpful. Appropriate use of IT also provides teachers with a way to cater for learner differences as it allows students with different abilities to learn at different paces.

6.30 School-based and class-based curriculum adaptations are possible ways in catering for the needs of low academic achievers as well as the more able students. Though the difficulties encountered by teachers and the workload of teachers are fully aware of, teachers are the most suitable persons to identify the needs and the progress of their students in school. Professionalism of teachers is crucial for the successful implementation of curriculum adaptation as learning experiences should be tailor-made for the individual students in order to maximize their potentials. However, merely trimming down the curriculum is not a desirable way to resolve the problem. Professional judgements have to be made on the selection of materials and the depth of treatment. Therefore, competent teachers are required to deal with the different mathematics abilities of students. Guidance, in-service courses and support for teaching students with learner differences offered by relevant bodies are invaluable.

6.31 Relevant and realistic contextual tasks should be carefully selected for students. Teaching strategies should be well-organized to provide a learner-centred situation to match the capabilities of students.

**Assessment Aspect**

6.32 In schools, more emphasis should be laid on the assessment of minimal competence as it is in line with the setting up of a minimal standard in the theory of individualized learning. Testing of unnecessarily complicated problems should be de-emphasized. So far as public examinations are concerned, we think that the existing practice laid down in the Tailoring Guide for the Secondary Mathematics Curriculum (1996) is feasible. Labeling effect is avoided in such a practice.
6.33 A wide range of assessment activities is recommended so that teachers can get adequate information to organize students’ learning experiences. Further details are provided in the subtitle “Assessment in the Mathematics Curriculum” on page 39.

Quality of Mathematics Teachers

6.34 Mathematics education is a key learning area of the school curriculum. Implementation of the mathematics curriculum relies very much on the supply of competent and well-prepared teachers who are able to realize the ideals of the mathematics curriculum during implementation. We opine that a good mathematics teacher should be a committed teacher who has strong motivation in promoting mathematics education, a reflective teacher who can always adapt himself to a fast-changing working environment, and a scholarly teacher who possesses sound understanding in mathematics, pedagogy of teaching and the needs of students.

6.35 In HK, qualified mathematics teachers are mainly provided by teacher education providers through pre-service and in-service teacher education programmes. The HKIEd starts to offer BEd degree programmes in 1998. It has a range of full-time pre-service Certificate in Education programmes for preparing primary and secondary teachers. The Institute also offers a range of full-time and part-time in-service programmes for the professional development of teachers in the early childhood, primary, secondary, technical and special education sectors.

6.36 Both HKU and CUHK offer BEd degree programmes as well as full-time and part-time post-graduate teacher education courses. Master of Education courses with specialty in mathematics education are also offered regularly. Other local institutions like the HKBU, OUHK, etc., offer in-service BEd degree programmes to school teachers while some overseas institutes also organize taught courses and distance learning courses for their BEd programmes through local agencies.

6.37 Currently, all tertiary institutions in HK do not offer any full-time BEd courses majoring in Mathematics. HKU is at present the only institute that has a part-time BEd course majoring in Mathematics for practising teachers.

6.38 A high proportion of primary mathematics teachers are non-degree holders and a large number of non-subject trained teachers are appointed to teach mathematics. On the contrary,

\[\text{CUHK will offer a full-time BEd course majoring in mathematics in September 2000.}\]
a majority of secondary mathematics teachers hold a bachelor’s degree in mathematics or related subjects (like science and engineering) and a significant number of secondary school teachers possess PGDE or PGCE qualifications.

**Recommendations**

6.39 A good mastery of teaching methods as well as a strong mathematical background are important traits for a mathematics teacher. Therefore, it is advisable to strengthen the subject specific component in BEd courses. On the other hand, training of teaching methods should be provided to practising teachers with BA/BSc degrees in mathematics.

6.40 It is desirable to have subject specialists to teach mathematics to students at the upper primary level or above. The primary mathematics teacher should preferably possess a BEd degree majoring in mathematics and the secondary mathematics teacher should be one with a strong mathematical background and suitable teacher training. We generally agree that an ideal qualification for a mathematics teacher, in the long-run, is a bachelor’s degree in mathematics or related discipline together with PGDE or PGCE.

6.41 Mathematics teachers should see the importance of life-long education for teachers. Collegiate exchange among mathematics teachers both within schools and in the mathematics education circle should be encouraged. The Curriculum Resources Centre of CDI at Tin Kwong Road, Kowloon and the Education Resources Centre of AID at Pak Fuk Road, North Point, HK provide places for teachers to share their teaching ideas and experiences. The teaching and reference materials published by both CDI and AID also provide good resources to teachers.

**Using Information Technology in Mathematics Education**

6.42 IT has become a fact of life and we have enjoyed many benefits from the advent of information and communication technology. We hold that our teachers should make good use of the technology to enhance teaching and learning while our youngsters should master the technology to exchange and process information, to think and work logically and to become more adaptive to the dynamically changing environment.

6.43 We agree that using IT in teaching and learning mathematics may bring about the following benefits:

(a) IT can enhance and extend mathematics learning experience, and encourage active student participation in exploratory and investigative activities.

(b) IT, when used as a tool, can support, supplement and extend teaching and learning
activities, such as:
• exercises and tutorial,
• charting and graphical analysis,
• simulation and modeling,
• information retrieval and handling, and
• data processing.

c) IT may lead to new teaching strategies and practices in classrooms such as providing students with an interactive learning environment for contextual and situation learning.

Recommendations
6.44 We propose the CDC Committee on Mathematics Education to further explore the uses of the following IT tools in teaching and learning mathematics at their particular level and subject(s):
• Calculators
• Multimedia educational software packages
• Graphing calculators
• Mathematics software tools
• Spreadsheets
• Internet

6.45 The future mathematics curriculum will focus on the effective use of information for problem solving - one of the principal reasons for studying mathematics. However, extensive use of the above IT tools may lead to de-emphasis of skills and trimming down of technicality. Therefore, we stress that mathematics should be taught in its own right and with its own educational objectives for the information age.

6.46 IT tools, in some cases, may provide shortcuts which are undesirable, especially in learning processes where mental development is needed for concept building. Therefore, we realize that IT should be cautiously used in classrooms and such messages and examples should be conveyed to mathematics teachers during teacher training sessions. Teachers should also act professionally towards choosing the most appropriate educational technology to benefit their students.

6.47 Effective use of IT in mathematics education depends on many factors. The IT education planners, curriculum developers and school administrators should be fully aware of the consequences of these factors and should address these issues tactfully in designing territory-wide implementation strategies and school-based plans:
• IT competency among mathematics teachers
- IT proficiency of students at various levels of schooling
- Teachers' knowledge and skills on strategies of using IT in classrooms
- Need for quality educational software and mathematics software tools
- Equity of access to IT equipment
- Level of technical and curriculum support within schools
Chapter 7

Summary of Recommendations

7.1 This chapter summarizes our recommendations made in the previous chapters. They are grouped into the following categories:

Chapter 4  Direction of Changes
Chapter 5  The Anticipated Mathematics Curriculum
Chapter 6  Implementation Strategies

Direction of Changes

7.2 Mathematics should be treated as an intellectual endeavour and a mode of thinking rather than a tool.
(paragraph 4.8)

7.3 The curriculum content of the primary and secondary mathematics curricula should be developed to materialize the aims of the curricula. Explicit statements for various aspects of the specific aims at the sixth form level and in each subject should be developed with reference to those aims of the primary and secondary mathematics curricula.
(paragraph 4.10)

The Anticipated Mathematics Curriculum

7.4 The mathematics curriculum should be designed according to a set of content-based learning dimensions so that learning objectives and students' progress can be structured and represented systematically from primary through secondary levels. However, it is not necessary to extend the use of dimensions in designing sixth form mathematics curriculum.
(paragraph 5.4)

7.5 The mathematics curriculum should be designed in such a way that mathematics learning progresses from concrete to abstract. The content in the mathematics curriculum should be arranged to let students get adequate prior experience with concrete objects before the formal treatment of mathematical concepts. Abstract concepts should also be backed up
by an abundance of mathematical and non-mathematical (daily-life) examples.
(paragraph 5.5)

7.6 HOTs should be incorporated into the content-based learning dimensions in designing the future mathematics curriculum.
(paragraph 5.6)

7.7 Teachers should be aware of those mathematical learning experiences, like social mathematics, mathematics appreciation, history of mathematics, which are not easily defined within the framework.
(paragraph 5.7)

7.8 Different levels should be considered as a continuous learning process. Objectives across various levels should be coherent and allow the development of mathematical concepts from concrete to abstract. The learning objectives and students' progress should be structured and systematically represented within and across levels.
(paragraph 5.14)

7.9 The acquisition of process abilities should be allied with the learning of mathematical content. Topics in the mathematics curriculum should be arranged into progressive learning dimensions so that students' cognitive development in mathematics can be enhanced in a more coordinated way and the relevance of students ensured.
(paragraph 5.15)

7.10 The mathematics curriculum should be developed as a whole, but it should be duly adjusted across different levels of learning to cater for the different abilities of students. Teaching too many topics in a single year and making learning fragmented should be avoided. Some measures like organizing bridging programmes should be taken to ensure that students of different abilities can follow. In the curriculum development process, a flow chart indicating the inter-relation of topics of different mathematics curricula at different levels is recommended to ensure continuity.
(paragraph 5.16)

7.11 Teaching strategies should be progressively changed through different levels of schooling, say from concrete to abstract, so as to cope with students' development. Teaching at P.1 should selectively adopt a thematic approach. Investigational work is encouraged at all the primary levels and should be continued in secondary schools.
(paragraph 5.17)
7.12 Teachers at different levels should be well informed on what is going on at the others. Measures should be taken to reinforce interflow between primary and secondary mathematics teachers.
(paragraph 5.18)

7.13 Children at the pre-primary level should not be formally assessed and no prerequisite academic knowledge should be expected of them on their admission to P.1. Heuristic methods of teaching should be adopted to help children foster an interest in learning mathematics and develop an inquiry mind. Thematic approach with integrated activities of learning should be used for the purpose of flexible curriculum integration.
(paragraph 5.22)

7.14 Mathematical concepts should be introduced according to the developmental stages of children, their interests and needs, by using simple language, and through learning activities which involve manipulation of objects and relate to children's experience.
(paragraph 5.23)

7.15 Children should be given the opportunities to explore, discover and develop mathematical ideas. Importance should be given to the process of learning mathematics rather than the outcomes of the activities. Flexibility should be allowed in planning teaching programmes and designing learning activities to cater for the learner differences, and incidental teaching should be adopted.
(paragraph 5.24)

7.16 Mathematical jargons like statistics, sum and difference, etc. which are beyond children's understanding should not be used. Over-teaching, premature exposure to mathematical knowledge and abstract concepts should be avoided as they may hamper children's development of learning mathematics.
(paragraph 5.25)

7.17 We do not agree to have curriculum differentiation in the years of general education. Instead, mathematics should be a subject studied by all students. To cater for the needs of different students at the upper end of the secondary schooling, the mathematics curriculum should be re-structured. A number of subjects, which may come from different combinations of modules and papers, should be designed for students who aim to further their studies in the mathematics related fields. Streaming students at an early age should be
avoided and opportunities for taking mathematics at the senior levels should be allowed for all students.
(paragraphs 5.29 – 5.31)

Implementation Strategies

7.18 We suggest the CDC Committee on Mathematics Education to consider our recommendations during the review of the Primary Mathematics Syllabus. At the same time, the content of the Secondary Mathematics Syllabus can be re-adjusted, if deemed necessary.
(paragraph 6.9)

7.19 The possibilities of setting minimal competence for mathematics at various stages of schooling should be explored.
(paragraph 6.15)

7.20 Assessment for high-stake purposes (such as placement and selection) should be played down to reduce over-drilling to students and minimize interruption to normal teaching and learning in schools. Assessment should be used as a means to collect feedback from students to improve teaching and learning.
(paragraph 6.16)

7.21 The needs of students at both ends of the ability scale are equally important. Attention should not be placed only on academically lower achievers. The needs of the more able students should also be catered for. More resources should be provided to strengthen the existing institution for the gifted and those related mathematics activities. However, means should be taken to prevent labeling effect.
(paragraph 6.18)

7.22 The mathematics curriculum should be flexible enough to cater for a wide spectrum of abilities of students. Measures, such as identifying the foundation and non-foundation components, allocating spare time periods, facilitating curriculum adaptation, can also be considered.
(paragraphs 6.22 and 6.24)

7.23 As learning content and objectives in secondary mathematics curriculum are arranged in terms of key stages instead of levels, teachers can judge for themselves their own sequences of learning units by basing on the needs of their students.
(paragraph 6.25)
7.24 According to the needs, interests and abilities of students, schools can adopt organizational arrangements, such as ability grouping, as well as instructional and curricular arrangements (such as remedial teaching, integrated learning and enrichment activities), in catering for learner differences. (paragraph 6.27)

7.25 Mathematics-related activities like mathematics clubs, quizzes, competitions, games, projects and workshops are good means both to cultivate the interest of students and to provide students with learning experiences through informal curriculum. (paragraph 6.28)

7.26 Active and purposeful learning activities, such as project work, worksheets, graded and optional class exercises, suggested leisure reading, that allows individualized ways in the construction of knowledge and appropriate use of IT provide teachers with ways to cater for learner differences. (paragraph 6.29)

7.27 Relevant and realistic contextual tasks should be carefully selected for students. Teaching strategies should be well-organized to provide a learner-centred situation to match the capabilities of students. (paragraph 6.31)

7.28 More emphasis should be laid on the assessment of minimal competence. Testing of unnecessarily complicated problems should be de-emphasized. A wide range of assessment activities is recommended for getting adequate information to organize students' learning experiences. (paragraphs 6.32 and 6.33)

7.29 It is desirable to have subject specialists to teach mathematics to students at the upper primary level or above. The primary mathematics teacher should preferably possess a BEd degree majoring in mathematics and the secondary mathematics teacher should be one with a strong mathematical background and suitable teacher training. An ideal qualification for a mathematics teacher, in the long-run, is a bachelor’s degree in mathematics or related discipline together with PGDE or PGCE. (paragraph 6.40)
7.30 Collegiate exchange among mathematics teachers both within schools and in the mathematics education circle should be encouraged.

(paragraph 6.41)

7.31 We propose the CDC Committee on Mathematics Education to further explore the uses of IT tools in teaching and learning mathematics at their particular level and subject(s).

(paragraph 6.44)

7.32 Extensive use of IT tools may lead to de-emphasizing of skills and trimming down of technicality. Therefore, mathematics should be taught in its own right and with its own educational objectives for the information age.

(paragraph 6.45)

7.33 We realize that IT should be cautiously used in classrooms and such messages and examples should be conveyed to mathematics teachers during teacher training sessions. Teachers should also act professionally towards choosing the most appropriate educational technology to benefit their students.

(paragraph 6.46)

7.34 The IT education planners, curriculum developers and school administrators should be fully aware of the consequences of the influence of mis-use of IT and should address these issues tactfully in designing territory-wide implementation strategies and school-based plans:

- IT competency among mathematics teachers
- IT proficiency of students at various level of schooling
- Teachers' knowledge and skills on strategies of using IT in classrooms
- Need for quality educational software and mathematics software tools
- Equity of access to IT equipment
- Level of technical and curriculum support within schools

(paragraph 6.47)
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Executive Summary of Research 1 –
Comparative Studies of the Mathematics Curricula
of Major Asian and Western Countries

This research was one of two related research studies commissioned by the Education Department on the mathematics curriculum. The other study was on the views of various sectors on the mathematics curriculum. The two studies constituted a coherent whole and their findings complemented each other. On the basis of the findings of both studies, recommendations were made in the other research report.

Background
In July 1997, an Ad hoc Committee on Holistic Review of Mathematics Curriculum was set up by the Curriculum Development Council (CDC) to conduct a holistic review of the mathematics curriculum in Hong Kong. Two supportive research studies were thus commissioned by the Education Department to provide inputs with sound academic principles and practical demand to the Committee in its work of reviewing the mathematics curriculum. It was anticipated that the analysis of the mathematics curricula of some major Asian and western countries and the strengths and weaknesses of Hong Kong students in comparison to students of other countries in this research would provide insight at the international level for local use.

Research questions and methodology
According to the objectives, the following research tasks and questions were set:
(a) To study the mathematics curricula of the major Asian and western countries with reference to their
(i) curriculum documents including aims and objectives, content and depth of treatment of each domain, modes of courses;
(ii) modes of assessment (both internal and external);
(iii) implemented curriculum including scale of implementation, problems encountered and actual effort paid by their students in studying mathematics; and
(iv) strengths and weaknesses of students in each country.
(b) To analyze the current mathematics curriculum of Hong Kong in comparison with and in contrast to overseas curricula in terms of
(i) aims and objectives;
(ii) modes of courses;
(iii) content and depth of treatment of learning areas (both described in the document and in real practice);
(iv) modes of assessment (both internal and external); and
(v) strengths and weaknesses of local students in mathematics.

(c) To make recommendations on the
(i) aims and objectives of the future mathematics curriculum;
(ii) modes of courses;
(iii) modes of assessment (both internal and external); and
(iv) learning areas that need to pay more attention and learning areas that need to pay less attention.

In this research, document analysis and synthesis of previous studies were carried out in investigating the worldwide trends in mathematics education. Over 100 local and worldwide research papers and articles on the current state of mathematics education were reviewed and analyzed, providing background information on the strengths and weaknesses of Hong Kong students for the two related research studies. The aims and objectives, the contents and the modes of implementation of the mathematics curricula of the major Asian and western countries were compared with a reference of more than 30 curriculum documents from Hong Kong and seven other countries. Moreover, in the light of the relevant results of the Third International Mathematics and Science Study (TIMSS), mathematics teaching and learning, the achievement and the attitudes of Hong Kong students towards mathematics were investigated.

**Curriculum trends and Hong Kong's mathematics education in the wider world**

Compulsory education was an important and challenging issue confronting all countries. On the one hand, we should go for a "mathematics for all" curriculum, and on the other hand, mathematics teaching should cater for individual needs. Different countries had carried out different mathematics curriculum reforms to cater for the varied needs of individuals, and these reforms were designed according to their own traditions and cultures. Recent literature on the current state of mathematics education revealed that contemporary mathematics curriculum reforms focused on the approach rather than content. The enhancement of ability rather than skill as one of the goals of mathematics education, the impact of hi-tech, and individual differences were identified as the three major issues in universal mathematics education.
Aims and goals of mathematics education

There were two complementary categories of aims: those furthering the development of society and those furthering the development of the individual. The educational goals of mathematics curricula in 10 countries were summarized and classified into 3 categories, i.e. practical, disciplinary and cultural. In line with the worldwide trend, the task group of the Hong Kong CDC/Examination Authority Joint Working Party considered mathematics not merely as a technical tool, but also as an intellectual endeavour and a mode of thinking. It was believed that mathematics played a central role in human culture in a more general context.

On analyzing the mathematics curriculum standards of different countries, it was found that at the turn of the century, most of them were going for an expanding goal rather than aiming simply for acquisition of mathematical knowledge and skills. Application of mathematics in realistic problems, attitude of using mathematics in daily life problems, confidence in using mathematics, communicating with mathematics and appreciation of mathematics were receiving more emphasis. Quality education instead of coping-with-exam-education was stressed. Mode of thinking and high order abilities were promoted. In some countries, mathematical awareness and mathematisation were stressed. The mathematical calibre was mentioned and the continuation of the mathematics curriculum between levels was highlighted.

The relationship between "product" and "process" in mathematics curricula was widely discussed. Recent literature pointed out that there was no dichotomy between conceptual understanding and acquisition of basic skills. Both content and process were interactive elements of the curriculum and understanding was gained through techniques. Basic skills formed the foundation for the enhancement of high order abilities and their utilisation.

Generally speaking, the revised S1-S5 mathematics curriculum (1999) in Hong Kong was in line with the worldwide trends. Process abilities were very much emphasized in the West while basic skills and content were stressed in Asian countries. The Hong Kong mathematics curriculum attempted to strike a balance between process and content. However, further investigation for better linkage of the process and content was needed.

Impact of Information Technology

Information technology changed the mode of getting information and learning. The literature suggested that the role of information technology in the mathematics curriculum should be considered. In the era of information technology, skills were de-emphasized and more room was left for development of concepts.
Upon analyzing the mathematics curriculum of Hong Kong, it was found that the impact of information technology was also addressed though nothing was said on when and how the calculator and computer should be used to assist mathematics learning.

**Individual differences**

Ways in dealing with individual differences and mixed abilities among students was one of the major problems of universal education. In the era of universal education, a major concern of curriculum developers was how to maintain flexibility in a curriculum in order to cater for individual differences of students. Curriculum differentiation, the introduction of options of enrichment and remedial teaching were suggested in various educational systems in their mathematics curriculum reform.

Hong Kong was probably the place with the least flexibility and choice in its curriculum. At the curriculum structure level and the implemented curriculum level, a number of models were proposed in previous research studies.

**Continuity of the Hong Kong mathematics curriculum**

Continuity and consistency throughout the primary and secondary levels of the mathematics curricula were issues of concern in Hong Kong. In particular, the curriculum structure at the senior secondary level needed urgent attention. It was commented that the current mathematics curriculum document was too bulky. A curriculum framework from an epistemological perspective, which was conducive to learning, needed to be developed.

**Content**

The literature revealed that cultural factors played an important role in the mathematics curriculum of various countries. Asian countries and western countries treated the mathematics curriculum with different emphases and approaches. In Hong Kong, skills in solving mathematics problems were emphasized, whereas in London, care was given to individual difference. Asian countries and regions put a lot of emphasis on measurement, such as units, perimeter, area & volume, much more so than western countries. Coordinate Geometry was introduced at the junior secondary level in Hong Kong as a connection between Geometry and Algebra whilst the coordinate system was introduced with different approaches in the primary years in USA and UK. In East Asian countries, a "canonical" curriculum is stipulated by the governments which is followed closely in schools.

In Hong Kong, the introduction of topics into the curriculum was on average 2 years earlier than the international median. Basic skills and computations were some of the strong points
of mathematics education in the Chinese society. Hong Kong, Mainland China and Korea stressed computation ability in early grades. Some topics, such as Fraction and Decimal, were introduced, dealt with thoroughly and completed in primary school, whereas in other countries, these topics were continued to be dealt with in secondary school.

Nonetheless, the mathematics curriculum of Hong Kong was generally in line with the worldwide trends. Basic mathematical knowledge, skills as well as process abilities were stressed. Mathematics "senses" and applications were emphasized. Attitudes and affective factors for mathematics learning were taken into account.

Assessment

The dilemma between the goals of schooling for education and for selection, the shift from standards to standardization, the difference between quality of learning and checklist of learning outcome and curriculum control were widely discussed in the literature. It was commented that Asian countries should not simply import curricula from western countries since they had different cultural backgrounds. Mere change in curriculum document was not enough. More room for the professional development of teachers for better mathematics teaching should be made. De-emphasis of grades and downplay of examination culture were essential.

The literature pointed out that Confucian education, rote learning and examination driven-ness are not necessarily equivalent to each other. However, it was repeatedly found that Confucian education was often hampered by examinations. Examinations under the Confucian Heritage Culture (CHC) addressed low level cognitive goals and were highly competitive. Excessive pressure was exerted on teachers and students.

Students' perception of mathematics and learning styles

Success in mathematics was seen as a ticket of success for the future. It was reported in the literature that students in Hong Kong perceived mathematics as the solving of problems by simple methods, as a school subject having definite answers, as a subject of computations, as a set of rules, as a subject that requires thinking and as a useful subject with some parts not easily applicable to daily life. Students tended to identify mathematics by its terminology and subject content.

The literature revealed that CHC students had strong preference of surface approaches to learning. Practice was valued. This could serve as a firm foundation for the enhancement of high level abilities as long as practice and memorization go hand in hand with understanding, which could be deepened with practices with variations. CHC learners
possessed cultural potential for the deepening of understanding by means of repetitive learning. The excellent academic performance of Chinese learners was due to the synthesis of memorizing and understanding which was uncommon in the West.

**Students' mathematics performance**

Hong Kong had the highest population density among the 41 countries joining the TIMSS. It was among the countries with the largest class size and the least experienced mathematics teachers. Its education system was centralized, and spending for basic education was the lowest. Despite such adverse conditions, Hong Kong students did extremely well in the TIMSS mathematics tests. They outperformed students in most of the other countries, coming fourth behind Singapore, Japan and Korea. The superiority of the Hong Kong students over those in other countries became more marked when they proceeded from grade four to grade eight.

Although Hong Kong outperformed the western countries in mathematics in the TIMSS, it was less impressive when compared to East Asian countries. Furthermore, there were still a lot of simple and essential concepts and skills that many Hong Kong students failed to master. They performed significantly worse in the TIMSS Performance Assessment where students were required to conduct some hands-on activities.

Examiners' reports in Hong Kong public examinations revealed that the major problems for Hong Kong students were their inadequacies in tackling problems and in their mental processes, rather than a lack of skills or basic knowledge. Students were generally weak in tackling non-routine exploratory questions. Many candidates were unable to make use of the information to solve problems. Logical reasoning and handling problems involving a variety of topics were weak.

**Students' attitudes**

Mathematics achievement was found to be closely related to affective variables, especially attitude towards mathematics and self-concept, which in turn were related to self and parental expectation. The literature showed a high correlation between students' interest in mathematics and their achievements. Based on the analysis of the relevant result of the TIMSS, it was found that Hong Kong students in general had high regard for mathematics as it was often perceived as a major subject. Hong Kong students in both grades 4 and 8 thought that it was important to do well in mathematics. This was further enhanced by the high value placed on constant practice and painstaking effort on the part of students which were significant features of the Chinese culture. However, Hong Kong students did not show exceptional interest in mathematics despite their high regard for it.
Primary school students' attitudes towards mathematics, as perceived by the teachers, were much better than the attitudes of those in secondary schools. In general, they did not believe in luck having anything to do with high achievement while teachers in Hong Kong tended not to believe in natural talent. Similar to other TIMSS countries, students in Hong Kong agreed that it took a lot of hard work to do well in mathematics. On the contrary, they considered sports or having fun less important compared to students in other parts of the world. Influenced by the achievement orientation of the Chinese culture, students made effort in their studies. However, not too much time in the school curriculum had been devoted to mathematics in Hong Kong compared to other parts of the world. This might be due to the relative emphasis on the second language in the Hong Kong curriculum.

The high achievement of Hong Kong students might have been achieved at the expense of other aspects of the development of the students. One possible trade-off of Hong Kong students' superior achievement in mathematics was their relatively low confidence in mathematics. Students did not display the corresponding level of positive attitudes and confidence towards mathematics. It was found that students' self-concept of learning mathematics dropped as they advanced through the grade levels.

One's confidence and self image were something that was reinforced by one's learned values, and if students were constantly taught to rate themselves low, they might internalize the idea and might result in really low confidence. Rigid and conformed modes of learning added extra hardship. The examination orientation reinforced the status quo. A feeling of alienation led to taking the mathematics class as boring, and paying great effort without arriving at desired outcomes generated frustrations. Attitude towards mathematics began to deteriorate and a lack of confidence finally led to giving up the subject.

The greatest concern was that Hong Kong students did not have much confidence in doing mathematics. In general, girls had a lower perception of their ability than boys. In sharp contrast to their high achievement, many students in Hong Kong did not think that they did well in mathematics. This may be a result of the Chinese culture on the virtue of humility or modesty, but it may also well be a result of the competitive examination system and a culture of lack of encouragement on the part of the teachers.

Classroom practice
Research revealed that Hong Kong students spent over one-third of their homework time on mathematics homework though the time spent had low correlation with their mathematics achievement. Classroom lecturing was the most common mode of instruction. In contrast
to western countries, which were more flexible in their use of textbooks, East Asian countries put a lot of emphasis on the textbooks. Most of the teachers in Hong Kong took textbooks as reference in preparing for their lessons, and they hesitated to perform curriculum tailoring. However, the textbooks in Hong Kong focused much of their attention on the performance expectations of "knowing" and "using routine procedures". The emphasis on examination further reinforced learning by rote.

The teacher
The teacher as "the authority in the classroom" phenomenon, which was often regarded as a hampering effect, and a mixture of authoritarianism and student-centredness in the CHC classroom was identified. It was evident that students hoped for a lively approach in teaching. Students valued a teacher who showed concern for them. The teacher was the key figure in student learning, especially for young students. The teacher's personality was a decisive factor of the students' liking of the subject. A good teacher was the most crucial factor leading to a good mathematics classroom as well as successful curriculum implementation. Teachers' ownership of the curriculum, sharing of educational goals and professionalism should be the prerequisites for curriculum change. The teacher's professionalism and development were at the heart of any curriculum reform and implementation. The TIMSS results revealed that the reform of mathematics curriculum in Japan took place in classroom teaching rather than mere revision of curriculum documents. More time was spent on applying mathematics concepts and thinking in the Japanese classrooms. The Japanese experience of "lesson study group" was worth exploring.

Conclusion
In a pluralistic and highly technological society, mathematics should be taught as a subject which possesses several different goals that reflect the diverse roles mathematics plays in the society. Students constructing their mathematical knowledge with an expanding goal is the world trend in mathematics curriculum reform. The essence of a curriculum lay in the approach rather than the arrangement of contents.

The current bulky mathematics curriculum needs to be re-examined to leave room for deep understanding and enhancement of high order thinking abilities but we should safeguard going for a watered-down curriculum. Adjustments made on the choice of teaching/learning activities and depth of treatment in individual topics are preferred rather than the simple addition and deletion of topics. In order to leave room for cultivating the motivation of student learning and the development of high order abilities, the scope and depth of the current curriculum content have to be re-considered.
Mathematics curriculum should aim for expanding goals. Mathematical knowledge, concepts, problem solving skills as well as abilities to discover and invent should be encompassed. Affective domains such as attitude and confidence, high order thinking abilities should also be taken care of. High order thinking should be addressed and cultural aspects of mathematics should not be overlooked in the future mathematics curriculum.

Due consideration should be given to both mathematical skills (content) and the process of learning (ability), as well as to the use of technology in mathematics teaching. Careful consideration should be given to examination orientations. The development of assessment of high order thinking is a world issue, but we should, at the same time, safeguard against having the curriculum driven by examinations. Of paramount importance was the promotion of student's interest in and understanding of the subject. As individual differences and mixed abilities are identified as major issues of concern for mathematics education in the next century, flexibility of the curriculum is necessary to cater for the above. All these new ideas stated above cannot be accomplished without the professionalism of mathematics teachers. Hence, a favourable curriculum should offer a far-reaching vision and also room for teachers to exercise their teaching at their best.
Appendix 2

Executive Summary of Research 2 – An Analysis of the Views of Various Sectors on the Mathematics Curriculum

The research focused on the views of students, parents, teachers, university lecturers, curriculum planners and human resources personnel in the commercial sector on the mathematics curriculum. Their views were solicited and the whole project progressed smoothly. The data arising from these studies were supported by results from another research\(^\text{(*)}\) commissioned by the Education Department.

**Background**

In response to the need for a review and possible reform in the mathematics curriculum in Hong Kong, the Ad hoc Committee for Holistic Review of the Mathematics Curriculum was set up in the Curriculum Development Council as it was thought that research on the strengths and weaknesses of the present curriculum and views of various stakeholders could not only provide useful information for curriculum planners in the development of a new curriculum, but could also generate first-hand data which could improve teaching and learning at the classroom level. It was with this intention that the Education Department commissioned the present research.

**Research questions and methodology**

In order to achieve the objectives, the following research tasks and questions were set:

(a) To study students' views at various learning stages on (i) their attitudes towards learning mathematics; (ii) their actual effort made in learning mathematics; (iii) their comments on the learning experiences; and (iv) the problems they face in learning mathematics.

(b) To study parents' views on the current school mathematics curriculum and their expectation for changes at various learning stages.

(c) To study teachers' views on (i) the current school mathematics curriculum at various learning stages; (ii) the problems they face in teaching; (iii) their expectation on future development; and (iv) the support they will need in the implementation of a new mathematics curriculum in future.

(d) To solicit views of various key stakeholders, including employers (from the human resources perspective) in various sectors of the employment field, educators of tertiary

\(^{(*)}\) The research was “Comparative Studies on the Mathematics Curricula of Major Asian and Western Countries”.

64
institutions/universities, etc., on (i) their general opinions on school mathematics education; (ii) the strengths and weaknesses of school-leavers in mathematics-related abilities; and (iii) mathematics-related abilities that need to be further developed.

(e) To compile suggestions made in another research(*) and feedback collected in consultation to make recommendations (with alternatives, and the pros and cons for each alternative) on (i) the overall aims of future school mathematics education, aims and objectives of the future school mathematics curriculum at various learning stages; (ii) the design and general layout of mathematics curriculum at various learning stages; (iii) modes of courses and modes of assessment; (iv) changes necessary to achieve the aims; and (v) strategic plans, both short-term and long-term, in implementing the recommendations.

Both quantitative and qualitative methods were utilised since the data to be collected are multifarious. Student and parent questionnaires were administered to a random sample of 10% of local primary and secondary schools. Open-ended questions were incorporated into the questionnaires. In-depth semi-structured interviews were conducted among students, university lecturers, curriculum planners and human resources personnel. Three basic research studies conducted by team members were also incorporated into the present one.

Results from another research
Review of literature revealed that individual differences and mixed abilities are major issues of concern for mathematics education in the next century. Flexibility of the curriculum is asked for to cater for the above. In a pluralistic and highly technological society, mathematics should be taught as a subject which possesses several very different goals that reflect the diverse roles mathematics plays in the society. Mathematical knowledge, concepts, problem solving skills as well as abilities to discover and to invent should be encompassed. The development of assessment of high order thinking is a world issue but we should, at the same time, safeguard against having the curriculum driven by examinations. Both the “content” and “process” of mathematics learning have to be taken care of. Cultivation of interest in learning mathematics is important too. In fact, international comparisons revealed that, though Hong Kong students performed well in mathematics tests, they lacked confidence in solving mathematical problems. How and when information technology could be used to enhance mathematics learning is another important issue that demands urgent exploration. In order to leave room for cultivating the motivation of student learning and the development of high order abilities, the scope and depth of the current curriculum content have to be re-considered. Again, international comparisons revealed that

(*) "Comparative Studies on the Mathematics Curricula of Major Asian and Western Countries" commissioned by the Education Department.
in Hong Kong, on the average, topics were taught one or two years earlier than in other countries. All these new ideas cannot be accomplished without the professionalism of mathematics teachers. Previous research did reveal that student learning in mathematics was greatly influenced by the teacher.

Students’ views
By the use of student questionnaires administered to nearly 9000 students, we see that students possessed a high regard for mathematics and preferred deep understanding rather than rote memorisation. They wished to know how formulas come about and are applied. They found interest in learning mathematics at a young age though such an interest declined and they found mathematics learning more and more difficult at higher grade levels. They experienced the greatest pressure from homework at Primary 6. Topics that involved tedious calculations were least welcome and word problems were thought to be difficult. Students hoped for liveliness and real life application both in teaching and in textbooks. Secondary school students felt that the syllabus at the junior secondary level was too fragmented and there was much overlapping of topics at Secondary 1 with those at primary levels. Senior secondary school students showed dissatisfaction with the whole senior secondary and sixth-form mathematics curriculum structure. They reflected that the syllabuses could not cater for their needs.

Interviews with 60 students further reinforced these findings. Students generally saw mathematics as a set of rules. At the same time they realised that the way one approaches a question and applies a formula and even one’s way of thinking were important if one was to solve mathematical problems. They saw homework as an important component of mathematical learning and so they hoped that teachers could provide them with sufficient exercises that provoked thinking. Their image of a good mathematics teacher was someone who is nice, lively, and who provides a variety of activities, offers clear, step-by-step explanation, allows time for students to think, checks frequently to see if students understand from time to time, explains how to approach problems and would not penalise weaker students. They reflected that their interest in learning mathematics was closely related to whether they could obtain a sense of success in solving mathematical problems. As found in the questionnaires, students disliked topics that involved tedious calculations and that were easy to make mistakes, impractical or difficult. Besides those discontents on the curriculum found in the questionnaires, students further pointed out that the curriculum was too packed in general and the case was even more serious at the senior primary level due to over-drilling for the Academic Aptitude Test. Some students found the use of computer software in teaching mathematics a waste of time.
Parents' views

The questionnaire was administered to over 6000 parents. Parents showed high regard for mathematics and held a positive view towards the mathematics curriculum as reflected in the parent questionnaire. Students' interest and understanding were their sole concern. Consistent with what was found among the students, parents hoped for clear explanation, motivation of interest, provision of exercises that provoke thinking and checking of students' understanding from time to time from the teacher. Parents showed great support to their children's learning of mathematics. They were willing to help with their children's learning and many of them, especially parents of students at lower grade levels, employed private tutors for their children. They believed in making effort and they relied on practices. To them, the major problems among students were carelessness and inability to interpret the questions. As their children moved up the grade levels, parents possessed less knowledge of the curriculum and there was a tendency of relying more on traditional ways of learning, like drilling with exercises. Parents generally held a negative feeling towards the Academic Aptitude Test and the quality of mathematics textbooks.

University lecturers' view

Interviews with university lecturers from nine departments gave us a picture of what they expected of our school mathematics curriculum. They were generally satisfied with students' standard and curriculum and saw the scores in public examinations as reliable. The demand on mathematics varied across departments and basically they could admit students of the appropriate calibre through competition of examination results. The only possible exception was the department of mathematics where they hoped to get students with a strong mathematical foundation and in reality this was not always possible. Some lecturers asked for breadth and some asked for depth in the school mathematics curriculum but in general, a firm foundation and a mathematical sense were of utmost importance. However, most of them did not possess much idea of the existing mathematics curriculum nor what was currently going on in school mathematics.

Views from human resources personnel

Interviews were conducted with human resources personnel in 5 enterprises. Most of the employers were satisfied with students' performance too. They saw language and attitude more important than mathematical knowledge. However, analytical power, problem solving skill and a sense of numbers were important in most of the careers.

Views of curriculum planners

Five curriculum planners were interviewed. The curriculum planner in the science area was satisfied with the mathematics curriculum, saying that it could provide the science subjects
with necessary mathematics tools. Interviews with mathematics curriculum planners at various levels revealed various problems of the existing mathematics curriculum. Lack of continuation from kindergarten to primary mathematics, inability to cater for individual differences at Certificate level, curriculum too packed in general, both Pure Mathematics and Applied Mathematics being too difficult were some of the problems they raised. Some suggested a shift from computation to conceptual understanding in the mathematics curriculum. They showed discontent on the over-emphasis of examinations in the community. The curriculum planners also urged for more communication between primary and secondary school teachers to improve coordination between these two levels.

**Teachers' view**

A total of 370 primary and 289 secondary mathematics teachers responded to the teacher questionnaire. Results revealed that mathematics teachers felt they possessed adequate mathematics knowledge to teach except for calculus and classical mechanics in the sixth-form where some teachers did not have sufficient confidence. Ability and motivation to learn were perceived as the major problems among the students in learning mathematics. Mixed ability was another key issue too. The students performed less well in those topics that involved tedious computation. As for the curriculum, most of the mathematics teachers who participated in the survey reflected that it was too bulky, lacked flexibility, was unable to cater for individual difference and to provoke thinking. The content was found to be dry too. Teachers had a tendency to tackle individual differences themselves and were not inclined to more systematic ways such as setting different assessment standards for different classes, where fairness was an issue of concern. Not many of the teachers incorporated information technology into their teaching at the moment and when they wanted to seek help in their teaching, collegiate exchange, their own school experience and textbooks would be their preferences. They seldom took the curriculum documents or seminars as a source of help. It is note-worthy that many primary school teachers did not have strong mathematics background and generally urged for a reduction of workload, in particular of non-teaching duties.

The above is consistent with what was found in the interviews with teachers which were conducted among 14 primary and 20 secondary mathematics teachers. They said that students were good at mechanical computation but weak in conceptual understanding and high order thinking. Students had a short attention span and at the secondary level different problems emerged, namely, students being passive, unable to take the initiative and not being serious about learning. Another serious problem was that they lacked a solid foundation. Almost all teachers pointed out that the existing mathematics curricula were too packed, too boring, impractical and unrelated to real life. They advised that continuity at all levels must
be secured. Content and level of difficulty should be rearranged with a strong epistemological and pedagogical foundation. If streaming is to take place at the senior secondary level, then opportunity for further mathematics studies at sixth-form must be offered as a viable option. The idea of a core and extended curriculum seems to be workable but we must let parents understand the rationale behind this notion. Teachers agreed that high order abilities should be addressed. The curriculum should be trimmed down to leave time and space for this to take place. Teachers generally showed high regard for information technology but they lacked guidance and support at all levels. Furthermore, they considered the use of information technology to be time-consuming. All in all, time was a big concern for teachers. Teachers needed more time to prepare teaching material. Suggestions from teachers included reduction in teacher-student ratio, class size and teaching workload, improvements to the crowded workplace, teachers' morale and social recognition of their profession.

**Conclusion**

The above results gave a clear picture that the current mathematics curriculum was well supported by various stakeholders though there are rooms for improvements. They cast great trust on the existing system. Both students and parents showed high regard for mathematics and they all opted for understanding rather than learning by rote. These are advantageous factors for the mathematics curriculum reform. Students, parents and teachers saw the basic skills as some of the strengths of the current curriculum and it can provide the students with a solid foundation. This is also reflected in the results in international comparative studies. Teachers also found the curriculum clear and easy to follow. We think that such strong points should be retained in the future mathematics curriculum innovation.

To go for “mathematics-for-all”, mathematics should no longer be taught in school just as a tool, but should be taught as a subject which possesses an expanding goal that reflects the diverse roles mathematics plays in the society. To maintain the interest of learning mathematics which young students already have, mathematics should be taught in a more lively and interesting way. So, the textbooks should include a variety of learning activities including real life examples and exercises that provoke thinking. The position and use of information technology in mathematics education is an issue of concern to different stakeholders but teachers showed hesitation in using it due to the lack of direction and guidance. We see that further developmental research is needed to explore how and when information technology can be used to make mathematics learning more effective.
Individual differences among the students, including their future needs as they enter different walks of life, is a major issue in the period of universal education. To cater for such needs, curriculum differentiation has to be considered which includes the reorganization of the senior secondary and sixth form curriculum structure. In order to address high order thinking and other process abilities, the curriculum should be enhanced to ensure continuation at different levels and to avoid overlapping and fragmentation. Unnecessary mechanical calculation and impractical topics should be removed. Thus, if there should be a trim-down of the content, it is only because we want to spare room for deeper understanding of the material rather than going for a water-downed curriculum. We need strong theoretical foundation to reorganise the content of the mathematics reasonably.

Any curriculum cannot be successfully implemented without the teacher. We see that the teacher should play an active and important role in the new curriculum. It should not be a document that is passed to the teachers for them to follow closely. On the other hand, the implementation of the new curriculum is demanding on the teacher's side. The teachers are expected to teach more lively to maintain students' interest and confidence in doing mathematics and to give them a sense of success. The teachers should also possess the ability to handle mixed-ability classes and cater for individual differences. They should address high order thinking and sense-making of mathematics and enhance problem solving abilities among the students. Mathematics teachers should consider the appropriate use of information technology in their teaching to make mathematics learning more effective. Teachers should widen their conception of mathematics and of mathematics learning too.

All these cannot be done without the upgrading of teacher professionalism. Teacher training and support are important. Guidance to teachers on various issues like curriculum tailoring and use of information technology is beneficial. Collegiate exchange among mathematics teachers both within schools and in the mathematics education circle should be encouraged. In particular, communication among mathematics teachers at primary and secondary levels should be strengthened. The Academic Aptitude Test is found to be disturbing and there is an urgent need for review. The emphasis on high order abilities should be reflected in assessments but every effort should be made to safeguard against backwash effects. We also see that reliable test items of high order abilities are not fully developed worldwide and research on it is necessary before the actual implementation of the idea. Different stakeholders, in particular university lecturers and parents, should be fully informed of the spirit of the new curriculum. Parents' understanding of the curriculum would guarantee meaningful support to their children's mathematics learning so that students can learn better.
Appendix 3

Aims of the Revised Primary Mathematics Curriculum

The primary mathematics curriculum aims to:

(a) stimulate the interest of children in the learning of mathematics;

(b) develop pupils' understanding and acquisition of basic mathematical concepts and computational skills;

(c) develop pupils' creativity, and their ability to think, communicate and solve problems;

(d) develop pupils' number sense and spatial sense, and their ability to appreciate patterns and structures of numbers and shapes;

(e) enhance pupils' lifelong learning abilities through basic mathematical knowledge.
Aims of the Revised Secondary Mathematics Curriculum

The secondary school mathematics curriculum continues the development of the learning of mathematics in the primary school. To enable students to cope confidently with the mathematics needed in their future studies, workplaces or daily life in a technological and information-rich society, the curriculum aims at developing students:

(a) the ability to conceptualize, inquire, reason and communicate mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts;

(b) the ability to manipulate numbers, symbols and other mathematical objects;

(c) the number sense, symbol sense, spatial sense and a sense of measurement as well as the capability in appreciating structures and patterns;

(d) a positive attitude towards mathematics and the capability in appreciating the aesthetic nature and cultural aspect of mathematics.
Appendix 5

Examples of Learning Dimensions

Hong Kong
- 5 dimensions in the Primary Mathematics Syllabus: Number; Algebra; Measures; Shape and Space; and Data Handling
- 3 combined dimensions in the proposed revised Secondary Mathematics Syllabus: Number and Algebra; Measures, Shape and Space; and Data Handling
- 4 stems in the Unofficial Mathematics Curriculum (designed by Mr. FUNG Chun-ip, HKIEd and Dr. WONG Ngai-ying, CUHK in December 1997): Numbers and Symbol Manipulation; Shape and Space; Measurement; and Handling Information

Australia
- 6 strands in the Australian mathematics curriculum: Working Mathematically; Space; Number; Measurement; Chance and Data; and Algebra

The United States
- 5 content strands, 3 mathematical abilities and 3 mathematical power together form a 3-dimensional structure in the US Department of Education's mathematics framework for the national assessment of educational progress:
  - 5 content strands: Number Sense, Properties and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics and Probability; and Algebra and Functions
  - 3 mathematical abilities: Conceptual Understanding; Procedural Knowledge; and Problem Solving
  - 3 mathematical power: Reasoning; Connections; and Communication

United Kingdom
- 4 attainment targets in mathematics in the UK national curriculum: Using and Applying Mathematics; Number and Algebra; Shape, Space and Measures; and Handling Data
Appendix 6
A Reference Grid for Learning Dimensions in the Future Mathematics Curriculum
Appendix 7
A Suggested Model of S.1 - 7 Mathematics Curriculum
(Based on the Current Academic Structure)
TOC